[6] H. Stapp, Phys. Rev. D3, 3177 (1971).
[7] J. Allaby et al., Preprint CERN 70 - 12 (1970).
[8] L. Ratner el al., Phys. Rev. Lett. 27, 68 (1971).
[9] K.G. Boreskov, A.B. Kaidalov, and L.A. Ponomarev, Paper delivered at Internat. Conf. on High-energy Physics, Oxford, 1972.
[10] E. Anderson et al., Phys. Rev. Lett. 19, 198 (1967).
[11] E. Anderson et al., Phys. Rev. Lett. 22, 102 (1969); 23, 721 (1969); P. Carlson et al., Phys. Lett. 35B, 502 (1970).

EQUATION OF STATE IN (4 - ε)-DIMENSIONAL ISING MODEL

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The main difficulty in the construction of a quantitative theory of phase transitions lies in the absence of free parameters, since the structure of matter near the critical point depends only on its symmetry, and the remaining parameters are determined by scale factors.

Wilson and Fisher [1] proposed to use the dimensionality of space as the free parameter. It is known that in 4-dimensional space the deviations from the Landau theory tend to zero near the transition point. In the space with 4 - ε dimensions the deviations are of order ε and perturbation theory can be used. Wilson [3] used this method to find, for the first time, the terms of the ε-expansion for the critical exponents. Although the series is obviously asymptotic, but through a fortunate coincidence the coefficients of ε and ε^2 turned out to be small, so that even at ε = 1 the first terms can describe the sum of the series fairly well. The discrepancy between the experimental and theoretical values of the indices is of the order of several per cent, and this gives grounds for hoping that the ε-expansion will work well also for other universal quantities, say for the equation of state.

Owing to scale invariance, the equation of state contains only one unknown universal function f(t):

$$H = (M/D)^{(\beta+\gamma)/\beta} f((T_c - T)(M/B)^{-1/\beta}) \qquad (1)$$
$$(f(1) = 0, \quad f(0) = 1).$$

Here H is the magnetic field, M the magnetic moment, and T the temperature. The non-universal constants B and D determine the spontaneous moment:

$$M_s = B(T_c - T)^{\beta}, \qquad (H = 0) \qquad (2)$$

and the moment in a strong field

$$M = D|H|^{\beta/(\beta+\gamma)} \qquad (T = T_c) \qquad (3)$$

β and γ are the usual critical exponents, the ε-expansion for which is of the form [3]:

$$\beta = (1/2) - (\epsilon/6) + (\epsilon^2/162) + 0(\epsilon^3), \qquad (4)$$

$$\gamma = 1 + (\epsilon/6) + (25/324)\epsilon^2 + 0(\epsilon^3). \qquad (5)$$

We have found the first three terms of the ε-expansion for the function f(t):

$$f(t) = (1 - t) + \frac{\epsilon}{6}\left[(3 - t)\ln(1 - \frac{t}{3}) - 2t\ln(2/3)\right] +$$
$$+ \frac{\epsilon^2}{648}\left\{9(9 - t)\ln^2(1 - \frac{t}{3}) + [150 - (50 + 36\ln\frac{2}{3})t] \times\right.$$
$$\left.\times \ln(1 - \frac{t}{3}) - (100\ln\frac{2}{3} + 36\ln^2\frac{2}{3})t\right\} + 0(\epsilon^3). \qquad (6)$$

The calculation method differs little from that used by Wilson [3], and we do not present the details of the calculations. Analogous calculations for the Heisenberg model with n-component spin will be published in a paper by G.M. Avdeeva.

The simplest form is possessed by the ε-expansion for the isocline $\phi(m)$ introduced in [4]. The isocline is the equation of state in terms of the variables H, M, and χ, where $\chi = (\partial M/\partial H)_T$ is the susceptibility in a finite field, i.e.,

$$H = \chi^{-(\beta+\gamma)/\gamma}\phi(M\chi^{\beta/\gamma}) \qquad (7)$$

accurate to ϵ^3 we have

$$\phi(m) = m - m^3 + (\epsilon^2/4)m^5 + 0(\epsilon^3). \qquad (8)$$

We normalize H and M in such a way that the coefficient of m^3 is equal to -1. It is interesting that formula (8) coincides exactly with formula (12) of the phenomenological theory [4] if ϕ_5/ϕ_3^2 in (12) is expressed in terms of β and γ, and the ε-expansion (4) and (5) is used for β and γ.[1)]

This equation of state was compared in [4] with experiment for the critical points of liquids. The agreement with experiment is on the order of several per cent, i.e., with the same accuracy as for the critical exponents β and γ.

We note that for the Heisenberg model in a Bose liquid, the ε-expansion of the isocline $\phi(m)$ will not be a polynomial in m, owing to effects connected with spin waves.

[1] K. Wilson and M. Fisher, Phys. Rev. Lett. 28, 240 (1972).
[2] A.I. Larkin and D.M. Khmel'nitsbii, Zh. Eksp. Teor. Fiz. 56, 2087 (1969) [Sov. Phys.-JETP 29, 1123 (1969)].
[3] K. Wilson, Phys. Rev. Lett. 28, 548 (1972).
[4] A.A. Migdal, Zh. Eksp. Teor. Fiz. 62, 1559 (1972) [Sov. Phys.-JETP 35, No. 4 (1972)].

[1)] There is a misprint in formula (34) of [4], where a factor 2 has been left out.