EFFECT OF COLLISIONS WITH PHONONS ON THE DINGLE TEMPERATURE FOR QUANTUM OSCILLATIONS

V.F. Gantmakher
Institute of Solid State Physics, USSR Academy of Sciences
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In his recent experiments on mercury, Palin [1] has shown that the Dingle temperature X (the collision broadening of the Landau levels), determined from the amplitude A of the de Haas - van Alphen oscillations

$$A \sim T H^{\frac{1}{2}} \exp[-2\pi^2 k(T + X)/\hbar\Omega], \qquad (1)$$

does not depend on the temperature T (H is the magnetic field, k is Boltzmann's constant, and Ω is the Larmor frequency). This result seemed at first glance to be very surprising, since it is known that for impurity scattering the Dingle temperature X_0 is connected with the collision frequency ν_0 by the relation

$$X_0 = \hbar\nu_0 / 2\pi k, \qquad (2)$$

and it might seem that a similar relation should connect $X_{e,ph} = X - X_0$ with the frequency of the electron-phonon collisions $\nu_{e,ph}$.

The result of [1] was explained theoretically in [2] on the basis of calculations of the oscillations of the magnetization in a system of electrons interacting with phonons. The result of the calculations could be interpreted in the following manner: The influence of the variation of the scattering probability with temperature, i.e., of the change of the imaginary part of the particle self energy in the electron-phonon system, is offset in (1) by the changes of the effective mass m (by the changes of the real part of the self energy), which enters in the argument of the exponential in (1) via Ω.

The purpose of the present article is to show that the result of [1] can be qualitatively explained also on the basis of "classical" calculations of the frequency $\nu_{e,ph}$ without making use of the renormalization of the electron mass. We consider this procedure to be all the more preferable since the result of the calculations in [2] pertain only to mercury, since the last stage of the calculation was carried out by numerical methods, using the parameters of this metal. In addition, these calculations pertain only to oscillations of the thermodynamic quantities, whereas variation of H causes also oscillations of the kinetic coefficients of the metal.

Let the initial electronic spectrum be quadratic. In a strong magnetic field $\vec{H} \parallel Oz$, it takes on the form of circular cylinders imbedded in one another, on which the occupied states lie inside the initial Fermi surface of radius p_f. Near the extremal section, the distance between the cylinders is $\Delta p = eH\hbar/p_f c$. In the quasiclassical approximation we can use the usual classification of the electronic states with the aid of the three momentum components and with a corresponding conservation law. It is then easily seen that near the Fermi sphere the electrons having a momentum

$$p_z \ll p_f \Delta p / q_T. \qquad (3)$$

along the field cannot be transferred by the collisions to another Landau cylinder ($q_T = kT/s$ is the momentum of the thermal phonon and s is the speed of sound).

The major role in the de Haas - van Alphen effect is played by those electrons which are located on the Landau cylinder at the instant when the latter

Circles - experimental plots of g(A/T) vs. T, taken from [1] (the size of the circle is a measure of the error). Crosses - result of subtracting from the experimental points the quantity $2\pi^2 k X_{e,ph}/\hbar\Omega$, which varies in proportion to $T^{3/2}$ ($X_{e,ph}(4°K) = 0.4°$). The upper temperature scale pertains to the upper pair of lines (marked by arrows).
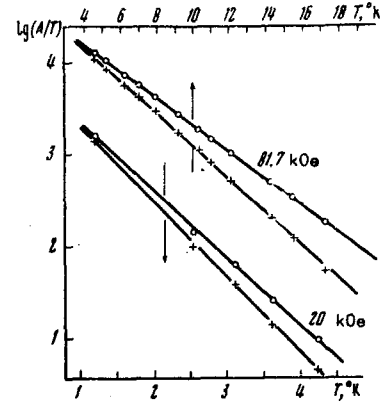
is tangent to the Fermi sphere, i.e., the electrons having

$$p_z^2 / 2m \lesssim kT, \qquad p_z \lesssim (2mkT)^{1/2}. \tag{4}$$

The condition (3) for this group of electrons takes the form

$$\hbar\Omega \gg (kT)^{3/2}/(ms^2)^{1/2}. \tag{5}$$

The last inequality practically coincides with the condition for the observation of quantum oscillations, $\hbar\Omega > 2\pi^2 kT$, since $ms^2 \simeq 0.1°K$.

Thus, all the de Haas - van Alphen electrons experience transitions only within the limits of the initial cylinder when they collide with the phonons. The probability of such transitions is

$$\nu_{e,ph}(p_z) = \frac{\Lambda^2}{4\pi\hbar^4\rho s} g \int q d^2 q \sum_{\pm} [\Phi_{\pm}(E, \epsilon) \delta (E_{p'} - E_p \mp \epsilon)], \quad \mathbf{p'} = \mathbf{p} + \mathbf{q}, \tag{6}$$

where Δ is the deformation potential, ρ is the density of the metal, E and $\epsilon = qs$ are the energies of the electron and the phonon, $g = (2\pi)^{-1} eH/p_f c$ is the degeneracy of the states on the cylinder, the δ-functions ensure satisfaction of the energy conservation law, the integration is over the surface of the cylinder, and the functions $\Phi_{\pm}$ are combinations of the phonon and electron distribution functions $n_{\epsilon} = [\exp(\epsilon/kT) - 1]^{-1}$ and $f_E = [\exp(E - E_f/kT) + 1]^{-1}$, respectively:

$$\Phi_+ = n_{\epsilon}(1 - f_{E+\epsilon}), \qquad \Phi_- = (n_{\epsilon} + 1)(1 - f_{E-\epsilon})$$

$\Phi_{\pm}$ enters in the phonon absorption probability and Φ_- is its emission probability.

Unlike the formulas for $\nu_{e,ph}$ without the field, the integration in (6) is over the surface and not the volume, and this leads in final analysis to a decrease of the degree of T; on the other hand, the field H enters in the factor g in (6).

The initial energy (4) of motion of the electrons of interest to us along the field is so small, that it can be neglected in the argument of the δ-function; only the phonon absorption need then be taken into account. Then

$$\nu_{e,ph} = \frac{\Delta^2 \Omega}{8\pi^2\hbar^4\rho v_f s} \int\int_0^{\infty} q dq_x d q_z \Phi_+(E, \epsilon) \delta\left(\frac{q_z^2}{2m} - s q\right) =$$

181

$$= \frac{1,5}{2\sqrt{2}\pi^2} \frac{\Delta^2}{\hbar^4 s^4 v_f \rho} (\hbar\Omega)(ms^2)^{1/2}(kT)^{3/2}, \tag{7}$$

where $v_f = p_f/m$, x is the direction of the cylinder surface and is perpendicular to the z axis, $q = (q_x^2 + q_z^2)^{1/2}$, and the coefficient 1.5 is the numerical value of the integral $\int_0^\infty e^x (e^{2x} - 1)^{-1} s^{1/2} dx$.

It is curious to note that the scattering in (7) cannot be regarded as elastic, since we cannot neglect the phonon energy qs in the argument of the δ-function.

The relation $\nu_{e,ph} \sim \Omega T^{3/2}$ explains the result of [1]. This result followed experimentally from the fact that, first, the slopes of the plots of $\ln(AH^{-1/2})$ vs. $1/H$ varied linearly with the temperature, and second, no deviation from linearity was observed in the plots of $\ln(A/T)$ vs. T. However, in view of the fact $\nu_{e,ph} \sim H$, the field H is cancelled out in the argument of the exponential $\exp(-2\pi^2 kX_{e,ph}/\hbar\Omega)$, so that the electron-phonon collisions can not affect the slopes of the straight plots of $\ln(AH^{-1/2})$ against H^{-1}. The presence of the term with $X_{e,ph}$ can be manifest only in a temperature dependence of the intercept on the ordinate axis.

As to the variations of A(T), if we were to have a direct proportionality of $\nu_{e,ph}$ to T, then the dependence of $\ln(A/T)$ on T would remain linear and the collisions with the phonons would lead only to a small change in the slope of the line, equivalent to a renormalization of the effective mass (this is practically impossible to discern in the experiments); therefore all the expected deviations from the linear dependence of $\ln(A/T)$ on T are due to deviation of the exponent 3/2 from unity.

Using the estimate given in [1] for the expected value $X_{e,ph} \simeq 0.4°$ at $T = 4°K$ and assuming $\nu_{e,ph} \sim T^{3/2}$, we have marked on the two plots of [1] the values of $2\pi^2 kX_{e,ph}/\hbar\Omega$ below the experimental points. The crosses obtained in this case fit the straight lines no worse than the initial points (see the figure). This means that the experimental accuracy is insufficient to determine the expected deviations of the plot from linearity.

Thus, in spite of the fact that in a strong magnetic field the frequency $\nu_{e,ph}$ of the electron-phonon collisions on the extremal section of the Fermi surface is $(\hbar\Omega)(ms^2)^{1/2}/(kT)^{3/2}$ times larger than without the field (the dependence given in (7) in place of the usual T^3), observation of these collisions with the aid of the amplitude of the quantum oscillations is a very difficult and possibly even impossible experimental task.

[1] C.J. Palin, Ph.D. Dissertation, Cambridge University, 1971.
[2] S. Engelsberg and G. Simpson, Phys. Rev. B2, 1657 (1970).