RESONANCE IN SUPERCONDUCTORS

M. Ya. Azbel'
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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A number of recent papers [1, 2] discuss magnetic surface levels in superconductors, analogous to those in normal metals (see [3]). We show in this paper that in superconductors there can exist a principally new type of bound states, due to a self-consistent interaction of excitations via a magnetic field H, and not present in mormal metals. In this case a band consisting of local levels is adjacent to the continuous spectrum at $|\epsilon| \leq \Delta$. Its width is of the order of the "interaction constant," which is proportional to H^2. The presence of such a spectrum leads to a resonance (see the end of the article).

To obtain the spectrum we turn to Gor'kov's equations for the wave functions ψ of the quasiparticles. We put

$$\psi = \begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} = \frac{\exp[i(p_x x + p_z z)/\hbar]}{\sqrt{2\xi\sqrt{\epsilon^2 - \Delta^2}}}\begin{pmatrix}\xi, & -\Delta \\ -\Delta, & \xi\end{pmatrix}\vec{\Phi}, \quad \xi = \epsilon + \sqrt{\epsilon^2 - \Delta^2}. \tag{1}$$

We then have in weak fields, in the approximation linear in the field (linear for the equation but not for its solution):

$$-\frac{\hbar^2}{2m}\vec{\Phi}'' - \mu_\perp\vec{\Phi} - \sqrt{\epsilon^2 - \Delta^2}\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\vec{\Phi} = \frac{a(y)}{\sqrt{\epsilon^2 - \Delta^2}}\begin{pmatrix}\epsilon, & -\Delta \\ \Delta, & -\epsilon\end{pmatrix}\vec{\Phi} = -\frac{\hbar^2}{2m}\mathbf{f}, \tag{2}$$

$$a(y) = \frac{ep_x}{mc} A(y), \quad \mu_\perp = \mu - \frac{p_x^2 + p_z^2}{2m}, \quad A = A_x(y) = \int_{-\infty}^{y} B(y')dy'.$$

Here ε is the energy, Δ the gap, μ the chemical potential, p the generalized momentum, $\vec{A}$ the vector potential, and $B = B_z$ the magnetic induction (this is just the quantity that enters in all the equations, i.e., the self-averaged magnetic field - see [4], with curl $\vec{B}$ - $(4\pi/c)\vec{j} \times \vec{B} = \text{curl } \vec{H} = 0$); the superconductor fills the half-space $y \leq 0$.

For any $|\varepsilon| \geq \Delta$, Eq. (2) has a solution that increases as $y \to -\infty$ and satisfies the boundary condition, which we shall write for concreteness in the form $\psi(0) = 0$. For $|\varepsilon| < \Delta$, the corresponding solution (continuous together with the first derivative) is

$$\phi_\sigma = k_\sigma^{-1} \exp(-i\sigma k_\sigma y) \int_{-\infty}^{0} \sin(k_\sigma y') f_\sigma(y') dy' + k_\sigma^{-1} \int_{-\infty}^{y} \sin[k_\sigma(y - y')] f_\sigma(y') dy', \tag{3}$$

$$\sigma = \pm 1, \quad k_\sigma = \sqrt{k_0^2 + \frac{2mi\sigma}{\hbar^2}\sqrt{\Delta^2 - \epsilon^2}}, \quad k_0 = \sqrt{2m\mu_\perp/\hbar^2}. \tag{4}$$

Let us write down Eq. (3) for $F_\sigma = \phi_+ - s\sigma\phi_-$ ($s = \pm 1$ determines the "upper" or "lower" split band). When $\kappa\delta \ll 1$ we have in the main approximation

$$F_+ = zs\,\mathrm{Im}\{\exp(ik_-y)\}\, k_0^{-1} \int_{-\infty}^{0} a(y') \sin(k_0 y') F_+(y') dy', \tag{5}$$

$$F_- = izs k_0^{-1} \{\mathrm{Re}[\exp(ik_-y)] \int_{-\infty}^{0} a(y') \sin(k_0 y') F_+(y') dy' + \tag{6}$$

$$+ \int_{-\infty}^{y} a(y') \sin[k_0(y - y')] F_+(y') dy' \},$$

$$z = \frac{4m\Delta}{\hbar^2\sqrt{2\Delta\epsilon'}}, \quad \epsilon = s(\Delta - \epsilon'), \quad \kappa = \frac{2m\sqrt{2\Delta\epsilon'}}{k_0\hbar^2}, \tag{7}$$

$$\delta = \int_{-\infty}^{0} a(y)\, dy / a_0, \qquad a_0 \equiv a(0).$$

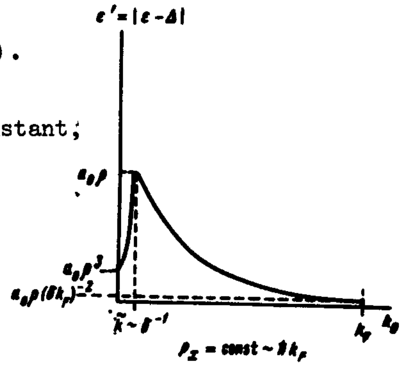

Fig. 1

From (5) we get $F_+ = D\,\mathrm{Im}[\exp(ik_-y)]$, where D is a real constant; and the local (at specified p_x and p_y) level (see Fig. 1)

$$\kappa = 8m^2\Delta s(\hbar^2 k_0)^{-2} \int_{-\infty}^{0} a(y) \sin(k_0 y) \sin\{y\mathrm{Re}k_-\} dy,$$

$$\epsilon' = (k_0\kappa\hbar^2)^2/(8m^2\Delta), \quad \kappa \sim \delta^{-1}\rho\left(1 + \frac{\rho}{k_0\delta}\right)(1 + k_0\delta)^{-2}, \tag{8}$$

$$\epsilon' \sim a_0\rho[(k_0\delta)^2 + (k_0\delta + \rho)^{-2}]^{-1}, \quad \rho = a_0\Delta\tilde{\epsilon}^{-2}, \quad \tilde{\epsilon} = \hbar^2/(2m\delta^2),$$

which exists (since $\kappa > 0$ by definition) only when sign a = sign s, i.e., at a definite sign of p_x, and corresponds to the ground state. The constant D is determined from the normalization of ψ.

The method is particularly lucid in the case when $\delta \to 0$, i.e., $a(y) = \delta a_0 \delta(y)$ ($\delta(y)$ is the delta function). Using the same model with $-L \leq y \leq 0$, it is convenient to trace the character of the spectrum. Going over to the Fourier representation in (2), we obtain

$$\begin{vmatrix} I_+ + \epsilon, & -\Delta \\ -\Delta, & I_- + \epsilon \end{vmatrix} = 0; \quad I_\pm = \frac{a_0(\epsilon^2 + \Delta^2)\delta}{\sqrt{\epsilon^2 - \Delta^2} \mp \xi_n}, \quad \xi_n = \frac{\hbar^2 n^2}{2mL^2} - \mu_\perp. \quad (10)$$

Equation (10) can be written in the form $F(\varepsilon) = a_0^{-1}$; its graphic solution is shown in Fig. 2. When $L \to \infty$, in the case when $|\varepsilon| > \Delta$, the continuous spectrum remains $(\varepsilon_{n+1} - \varepsilon_n \to 0)$ and Eq. (1) determines essentially I_+ or I_-, whereas when $|\varepsilon| < \Delta$ there appears a local self-consistent "zero-sound" level (since a sum is obtained over all k_n), corresponding to an interaction via the magnetic field.

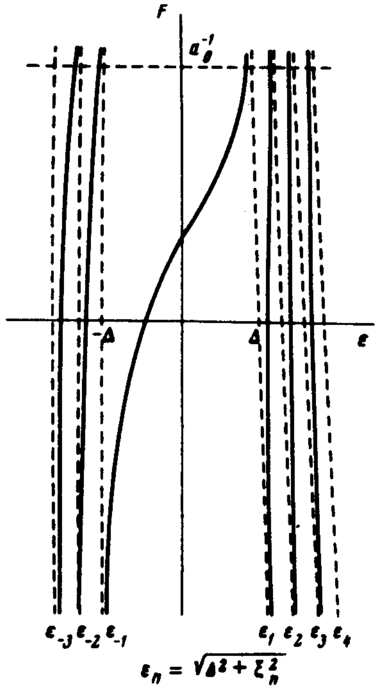

Fig. 2

The static current density

$$\mathbf{j} = \frac{4e}{m} \sum_{\epsilon > 0} \mathrm{Re}\{\psi_+^*(\mathbf{p} - \frac{e}{c}\mathbf{A})\psi_+\} \mathrm{th} \frac{}{2T}$$

receives from the local levels a contribution of the order of $\kappa\exp(2\kappa y)$, which is cancelled, however, by the continuous-spectrum states located near Δ, so that $\vec{j}$ has the usual form. (A slowly-damped $\vec{j}$ might appear when account is taken of the de dpendence of the quasiparticle reflection coefficient on the angle of their incidence on the surface, when small k_0 are separated.) At resonance, the local levels are separated, and this gives rise to current-density and magnetic field components that penetrate into the metal, are oscillatory in k_0^{-1}, but are weakly damped (in κ^{-1}).

Let us determine the density $\nu(\varepsilon')$ of the local-state band: $\nu(\varepsilon') = \iint \delta[\varepsilon' - \varepsilon'(k_0)] \times dp_x dp_z/h^2$. When $k_0 = \tilde{k} \sim \delta^{-1}$, when the entire wave is in the skin layer, $\varepsilon'(\tilde{k})$ (see Fig. 2) is maximal, and $\nu(\varepsilon')$ has a singularity. The density of states has a singularity (jump) also at the boundary of the spectrum with respect to k_0, at $k_0 = k_F = \sqrt{2m\mu}/\hbar$. Finally, the singularity of the density of states may be connected with the nonspecular character of the reflection starting with certain not-too-small $k_0 \sim k_1$. The singularity of $\nu(\varepsilon')$ denotes the possibility of a resonance in a superconductor placed in a constant magnetic field and in a weak high-frequency field $H_1 << H$, at a frequency $\hbar\omega = \varepsilon'(\tilde{k})$ ($k = k_F$ apparently corresponds to diffuse reflection, thereby excluding resonance), and anomalies in the high-frequency characteristics when $h\omega \sim \varepsilon'(k_+)$.

The condition for the applicability of the foregoing theory is $\kappa << \delta^{-1}$, i.e.,

$$\rho << \rho_k, \quad \rho_k = (1 + k_0\delta)^2. \quad (9)$$

When $\rho \sim \rho_k$ the next local levels split away from the gap, so that the quasiclassical approach holdes when $\rho >> \rho_k$ (ϕ attenuates more rapidly than a); calculations for this case, at any a_0/Δ, are given in [2]. The appearance of the next band of local states occurs at $k_0\delta \sim 1$ (the entire wave is in the skin layer), when $\rho_k \sim 1$. The origin of such a parameter is clear already from (2) when this equation is written in relative coordinates $y = \delta\gamma_1$, if

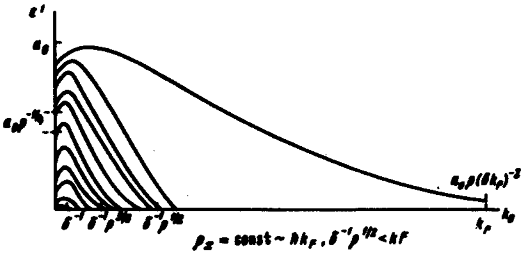

Fig. 3

account is taken of the fact that max $\{\sqrt{\varepsilon'}, a_0/\sqrt{\varepsilon'}\} \geq \sqrt{a_0}$, so that $\phi \sim \exp(i\kappa_1 y_1 + \kappa_2 y_1)$, $\kappa_1 - i\kappa_2 = [(k_0\delta)^2 - i\rho^{1/2}]^{1/2}$; the value of κ_2 determines the situation. When $k_0\delta \sim 1$ and $\rho >> 1$ the number of levels and the distance between them are of the order of $\rho^{1/4}$ and $a_0\delta^{-1/4}$ respectively - see Fig. 3. (We note that $\rho \sim H$, and that $\rho \sim 1$ is reached when $a_0 \sim \varepsilon^2/\Delta \sim \Delta(\delta^2 k_F/\xi_0)^{-2}$, ξ_0 is the pair dimension, i.e., when $H \sim ch\xi_0(e\delta^5 k_F^2)^{-1} \sim 10^{-6} - 10^{-4}$ Oe). With increasing k_0, the number of leveos decreases, and when $k_0 \sim \delta^{-1}\rho^{1/2}$ there remains one of the levels calculated above (if $\rho^{1/2} < \delta k_F$ and the reflection diffuseness that smears out the local levels does not become appreciable at smaller k_0). When H increases, all the new bands move away from the bottom of the gap, and all the curves of Fig. 3 move apart and shift upward.

Calculation of the density of states, the assessment of its singularities and of the possibility of resonance and analogous to those of the preceding case. The most convenient, apparently, are observation of resonance at $k_0 \sim \delta^{-1}$, at the frequencies $\hbar\omega \sim n a_0 \rho^{-1/4}$, $n = 1, \ldots < \rho^{1/4}$ at a natural level width $\hbar/\tau$, which is small compared with the distance $a_0\rho^{-1/4}$ between levels. In fields $H \sim 1$ Oe, this corresponds to $\omega \sim 10^9 - 10^{11}$ sec^{-1} and $\tau > 10^{-8} - 10^{-10}$ sec.

Inasmuch as the local levels correspond to finite motion of the quasiparticles and to their collisions with the surface, a study of the resonance yields information on the character of quasiparticle reflection [5]. When $\kappa\delta < 1$ the planarity of the surface is extremely important, for in the two-dimensional case ε' becomes exponentially small relative to $(\kappa\delta)^{-1}$, and accordingly the limitation $\varepsilon' >> \hbar/\tau$ becomes exceedingly stringent, while in the three-dimensional case, when $a = a(x, y, z)$, there are no local levels at all.

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