NARROWING OF DIFFRACTION CONE AND SLOPE OF POMERANCHUK TRAJECTORY

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Recently a group at the High Energy Laboratory of the Joint Institute of Nuclear Research (Dubna) obtained with the accelerator of the Institute of High Energy Physics (Serpukhov) very interesting data on the narrowing of the cone in elastic pp scattering [1]. Considering the region of very low $|t| = \kappa^2 \leq 0.1$ (GeV/c)^2, in which

$$\ln(d\sigma/d\kappa^2) = C - b(\xi)\,\kappa^2$$

the authors of [1] measured the coefficient $b(\xi)$ as a function of the energy (of $\xi = \ln(E_{lab}$ GeV)) in the region 10 GeV $\leq E_{lab} \leq$ 65 GeV.

We consider here this dependence theoretically with allowance for the contributions of several poles and branch points. We shall show that the branch points hardly change the effective slope $2\alpha'_{eff} = db(\xi)/d\xi$. For pp scattering, the value of α'_{eff} should be close to the slope $\alpha'_P(0)$ of the Pomeranchuk-pole trajectory ($\alpha'_{eff} - \alpha'_P(0) \lesssim 10^{-2}$), i.e., the experimental data are incompatible with the assumption that $\alpha'_P(0) = 0$. As the time, for πN scattering (and particularly for $\bar{p}p$ scattering), the effective narrowing of the cone should be still smaller [2] even for $E_{lab} \sim 65$ GeV, and its measurement calls for a high accuracy[1].

For very small κ^2 it suffices to take into account the contributions of the Regge poles $M_a^{(1)}$ and of the first branch points (double rescattering $M_{ab}^{(2)}$ [6, 7]). For the Pomeranchuk trajectory,

$$-iM_P^{(1)} = \gamma_P e^{-\lambda_P \kappa^2} \approx \gamma_P(1 - \lambda_P\kappa^2),$$

$$-iM_{PP}^{(2)} = -(\gamma_P \gamma_{PP}/4\lambda_P)\, e^{-\lambda_P\kappa^2/2} \approx -(\gamma_P\gamma_{PP}/4\lambda_P)\left(1 - \frac{\lambda_P\kappa^2}{2}\right),$$

and for any other trajectory

$$-iM_a^{(1)} = \eta'_a \epsilon_a \gamma_a e^{-\lambda_a\kappa^2} \approx \eta'_a\epsilon_a\gamma_a(1 - \lambda_a\kappa^2),$$

$$-iM_{aP}^{(2)} = -\eta'_a\epsilon_a \frac{\gamma_a\gamma_{aP}}{\lambda_a + \lambda_P}\exp\left[\frac{-\kappa^2}{\lambda_a^{-1} + \lambda_P^{-1}}\right] \approx \frac{-\eta'_a\epsilon_a\gamma_a\gamma_{aP}}{\lambda_a + \lambda_P}\left(1 - \frac{\kappa^2}{\lambda_a^{-1} + \lambda_P^{-1}}\right),$$

где $a = P', \omega, \rho, A_2$ и т. д., $\epsilon_a = \dfrac{1}{E^{1-\alpha_a(0)}}$, $E = E_{лаб}$ в Гэв, γ_P, γ_a

γ_P, γ_a and γ_{PP}, γ_{aP} are quantities with the dimension of length squared and determine the residues at the poles and the contributions of the rescatterings at the Pomeranchuk pole ($\gamma_{PP} = \gamma_{aP} = \gamma_P$) in the "optical" or "eikonal" approximation [8]),

$$\lambda_a = R_a^2 + \alpha'_a(0)\left(\ln \xi - \frac{i\pi}{2}\right), \quad a = P, P', \omega \text{ и т. д.},$$

with R_a^2 the parameters that determine [7] the dependence of the residues on κ^2, and a

$$\eta'_a = \sigma_a + i\left(\operatorname{ctg}\frac{\pi}{2}\alpha_a(0)\right)^{\sigma_a},$$

where $\sigma_a = \pm 1$ is the signature ($\eta'_a = i\eta_a$, where η_a is the usual signature factor at $t = 0$).

Recognizing that $d\sigma/d\kappa^2 = 4\pi\{|M|^2 + |N|^2\}$, where $M = M_P + \sum_{a \neq P} M_a$, $M_a \approx M_a^{(1)} + M_{aP}^{(2)}$ is the amplitude without spin flip, $N \approx (\kappa/2m_N)\sum_a i\eta'_a\epsilon_a\gamma_{a1}$ is its spin-flip part (accurate to terms proportional to κ^2), and that $\gamma_{PP} \sim \gamma_P < 4\lambda_P$, $\epsilon_a \ll 1$, $a \neq P$, we obtain, taking into account the terms linear in κ^2:

$$\frac{1}{2} b(\xi) = R_P^2 + \frac{\gamma_{PP}}{8} - \Delta(E) + \alpha'_P(0)\ln E + \sum_{a \neq P} \frac{\sigma_a C_a}{E^{1-\alpha_a(0)}}, \tag{1}$$

where[2]

[1] For pp scattering, $\alpha'_{eff} \sim 0.1 - 0.2$ at $E_{lab} \sim 65$ GeV. The reason for this [3 - 5] is that the P' and ω trajectories yield a narrow angular distributions and rapidly fade out with increasing E_{lab}. Their contributions almost cancel out in the pp case, and are additive in t the pp case. In the πN case the ω trajectory makes no contribution at all.

[2] We shall henceforth neglect terms of the type $i\pi\alpha'_a/2(R_a^2 + \alpha'_a\xi)$ compared with unity.

$$C_a = \frac{\gamma_a}{\gamma_P}\left\{R_a^2 - R_P^2 + (\alpha'_a - \alpha'_P)\xi + \gamma_{aP}\left(\frac{R_P^2 + \alpha'_P\xi}{R_a^2 + R_P^2 + (\alpha'_a + \alpha'_P)\xi}\right)^2\right\}, \tag{2}$$

a

$$\Delta(E) = (4m_N\gamma_P)^{-2}\left|\sum_a \frac{\eta'_a\epsilon_a\gamma_{a1}}{E^1 - \alpha_a(0)}\right|^2\left(1 + \frac{\gamma_{PP}}{2(R_P^2 + \alpha'_P\xi)}\right)$$

is the small contribution of the spin-flip amplitude. The branch points PP make a small constant contribution to (1), $\gamma_{PP}/8$ ($\sim$ 0.1 of R_P^2), and the remaining aP branch points change C_a only little (the last term in (2)).

The values of the parameters γ_a, γ_{a1}, R_a^2, $\alpha_a(0)$, and $\alpha'_a(0)$, corresponding to the experimental data [2] on $d\sigma/dt$ and σ_{tot} in the region $E_{lab} \sim 10 - 25$ GeV are listed in the table. For N, kN, NN-$\bar{N}$N scattering it is sufficient to take into account the P, P', ω, ρ, and A_2 trajectories. Assuming $\gamma_{PP} \simeq \gamma_{aP} \simeq \gamma_P$, $\alpha'_{P'}(0) = \alpha'_P(0)$, and $\alpha_\omega(0) \simeq \alpha_\rho(0) \simeq 1/2$, we obtain

$$\frac{1}{2}b_{\pi\pm N}(\xi) = R_P^2 + \frac{\gamma_P}{8} - \Delta_{\pi N} + \alpha'_P(0)\xi + (C_{P'} \mp C_\rho)e^{-\xi/2}, \tag{3}$$

$$\frac{1}{2}b_{NN}(\xi) = R_P^2 + \frac{\gamma_P}{8} - \Delta_{NN} + \alpha'_P(0)\xi + [C_{P'} + C_{A_2} \mp (C_\omega + C_\rho)]e^{-\xi/2}$$

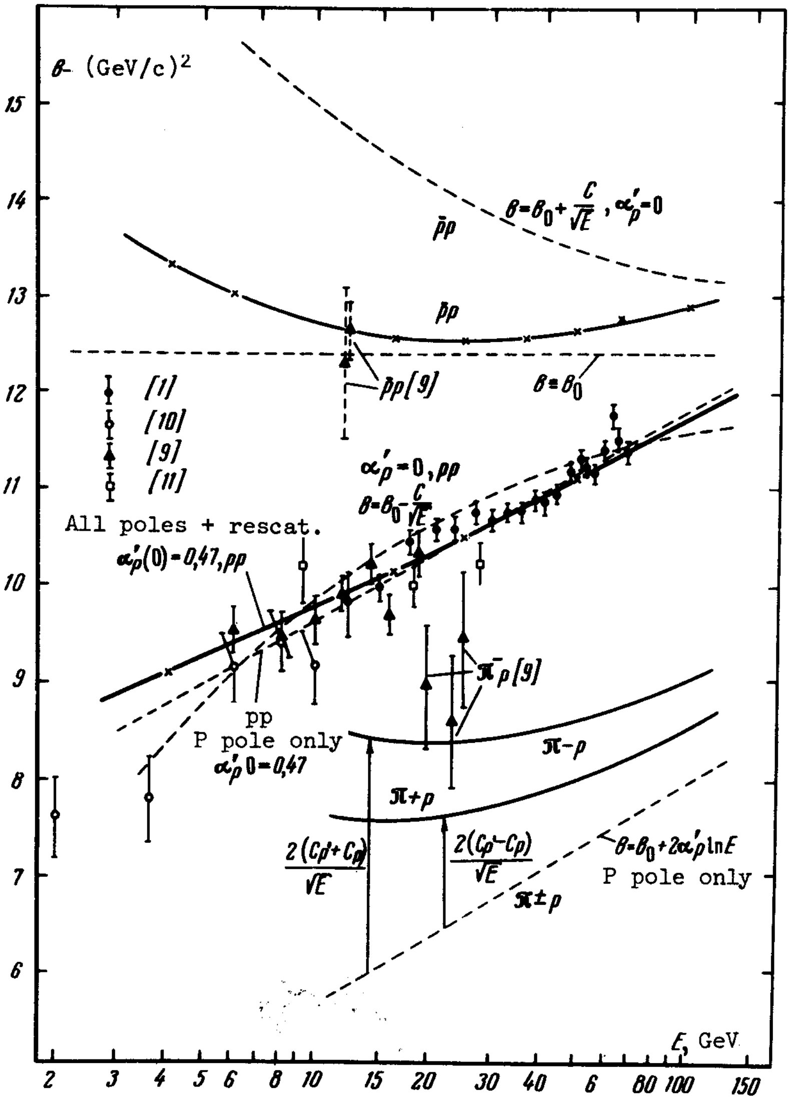
где

$$\Delta_{NN} = \left|\frac{\gamma_{P1} \mp (1-i)(\gamma_{\omega 1} + \gamma_{P1})e^{-\xi/2}}{4m_N\gamma_P}\right|^2\left[1 + \frac{\gamma_P}{2}(R_P^2 + \alpha'_P\xi)^{-1}\right]$$

and $\Delta_{\pi N}$, defined in the same manner but with $\gamma_{\omega 1} = 0$, are neglegibly small (since $m_N^2\gamma_P \sim 2$, and $\gamma_{P1} \sim \gamma_{\rho 1}e^{-\xi/2}$ are small). The upper sign pertains throughout to the cases of the scattering of particles (π^+, k^+, p) by protons, and the lower to antiparticles. The first term

Values of parameters for πN and NN-$\bar{N}$N
(α'_a, γ_a, γ_{a1}, R_a^2 - in $(GeV/c)^{-2}$).

	a	$\alpha_a(0)$	$\alpha'_a(0)$	γ_a	γ_{a1}	$R_a^2 = R_{a1}^2$
πN	P	1	0.5	2.42	0	1.05
	P'	0.5	0.5	2.60	0.3	4.20
	ρ	0.45	1.0	0.583	-4.6	3.20
NN - $\bar{N}$N	P	1	0.5	5.25	-0.26	3.07
	P'	0.5	0.5	6.63	2.00	4.20
	ω	0.5	0.8	3.51	3.20	4.80
	ρ	0.45	1.0	0.37	-3.38	3.20
	A_2	0.40	0.8	0.08	-0.52	3.20



in (3) is constant within 1%, so that $b(\xi)$ is the sum of a linear function $b_0 + \alpha'_P\xi$ and an exponentially decreasing one. A plot of $b(\xi)$ for the πp and pp cases is shown in the figure for $\alpha'_P(0) = 0.47\ (\text{GeV}/c)^{-2}$ and $\alpha'_P(0) = 0$. The last term in (3) changes the effective slope

$$\alpha'_{eff} = \frac{db(\xi)}{2\,d\xi} = \alpha'_P(0) - \frac{C_{P'} + C_{A_2} \mp (C_\omega + C_\rho)}{\sqrt{E}}, \tag{4}$$

where $C_\omega = C_{A_2} = 0$ for the πN case.

According to the table, in this case $C_{P'}/2 = 1.8$ and C_ρ is small, so that if $\alpha'_P(0) = 0.5$, then $\alpha'_{eff} \simeq 0.5 - 1.8/\sqrt{E}$ (in $(\text{GeV}/c)^{-2}$), i.e., $\alpha'_{eff} = 0$ in the region $E_{lab} \sim 20$ GeV and $\alpha'_{eff} = 0.25\ (\text{GeV}/c)^{-2}$ at $E_{lab} \sim 60$ GeV.

In the NN-N̄N case the parameters listed in the table are only tentative, since the complete reduction of the experimental data with allowance for the branch points has not been completed. This table gives $C_{P'} \simeq 2.80$, $C_\omega \simeq 2.50$, $C_\rho \simeq 0.78$, $C_{A_2} \simeq 0.04$, which leads to a negligibly small value of the last term[3] in (3) and (4) (yielding $\alpha'_{eff} - \alpha'_P(0) \leq 10^{-2}$). The experimental data for pp [1], as seen from the figure, agree well with the value $\alpha'_P(0) = 0.47$ $(GeV/c)^{-2}$.

The assumption that $\alpha'_P(0) = 0$ leads to a sharp discrepancy with the experimental data. Indeed, if we use in this case the values of the parameters from the table (but with $R_P^2 = 4.84$ $GeV/c)^{-2}$), then we get for $b(\xi)$ an almost constant value, $b(\xi) \approx 11.0$, which differs from the experimental data in the figure at large and small E by more than two standard deviations. Since formula (1) yields in the general case $b = b_0 + 2\alpha'_P(0)\xi + 2v/\sqrt{E}$, where $v_{pp} = C_{P'} + C_{A_2} - (C_\omega + C_\rho)$ and $v_{\bar{p}p} = C_{P'} + C_{A_2} + C_\omega + C_\rho$, the question can be stated in more general form: it is possible, when $\alpha'_P(0) = 0$, to choose the constants b_0, C_P, and $C_\omega + C_\rho$ so as to imitate the growth of the curve on the figure with the aid of the last term of $b(\xi)$. It is most convenient for this purpose to assume[4] $C_{P'} + C_{A_2} = 0$. Denoting $C = C_\omega + C_\rho$, we then obtain $b(\xi) = b_0 \pm 2C/\sqrt{E}$ when $\alpha'_P(0) = 0$, where the upper and lower signs pertain to the cases of pp and $\bar{p}p$ scattering, respectively. The best agreement for pp (see the curve on the figure) is obtained when $b_0 = 12.4$ and $C = 4.0$. But in this case we obtain for $\bar{p}p$ scattering the unreasonably large values $b(\xi) \sim 15$ $(GeV/c)^{-2}$, which is much larger than the experimental values [9] $b(\xi)$ 12 - 13 at $E \sim 12$ GeV. Thus, the value $\alpha'_P(0) = 0$ is apparently incompatible with all the experimental data.

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[3] It determines the difference between the dashed straight line $b = b_1 + \ln E$ and the curve $b = b(\xi)$ for the case pp in the figure.

[4] According to (2), negative $C_{P'}$ and C_{A_2} (which are even more convenient for the description of the data of [1] than zero values), are impossible when $\alpha'_P(0) = 0$, even when $R^2_{P'} = R^2_{A_2} = 0$.