SECOND CRITICAL CURRENT OF BULKY SUPERCONDUCTORS

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The occurrence of electrical resistance in bulky superconductors of the first kind takes place at the Silsbee point, i.e., at a current value J_{c1} such that its magnetic field reaches the critical value H_c somewhere in the sample. For a cylindrical wire, $J_{c1} = cH_cR/2$, where R is the radius of the wire. When $J > J_{c1}$, however, the sample does not become fully normal. A region of layered intermediate state remains in its interior (see [1, 2]), and the dimension of this region decreases with increasing current.

Clearly, there should exist a second critical current J_{c2} such that when $J > J_{c2}$ the sample becomes fully normal. In other words, when the current is decreased, the superconducting order parameter first appears in the sample at the point $J = J_{c2}$. In order that the usual picture of the transition from the superconducting state to the normal one via the intermediate region to occur with increasing current, the inequality $J_{c2} > J_{c1}$ must hold.

We shall calculate in this paper the critical current J_{c2} at temperatures close to the superconducting-transition critical temperature of bulky pure superconductors of the first kind. It turns out here that if the temperature is sufficiently close to critical, then the ratio J_{c2}/J_{c1} in bulky superconductors becomes smaller than unity. The character of the destruction of the superconductivity by the curent changes in this case.

1. To determine J_{c2}, we calculate the contribution made to the conductivity of a normal metal by fluctuations of the superconducting order parameter ψ with allowance for the magnetic field of the current. The value of the sample current at which a sharp increase of the fluctuation conductivity takes place is indeed the critical current J_{c2}.

The calculation of the fluctuation conductivity is analogous to the corresponding calculations made without allowance for the magnetic field of the current [3, 4]. We shall follow the method proposed in [4].

The fluctuation current through the sample is $J' = QE$, where

$$Q = -\frac{1}{LT}\lim_{\omega\to 0}\frac{1}{\omega}\operatorname{Im}\int_0^\infty dt\, e^{i\omega t}\langle \dot{\bar{j}}_z(0)\,\bar{j}_z(t)\rangle, \tag{1}$$

$$\bar{j} = \int dV j'(r),\quad j' = -\frac{ie}{m}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{4e^2}{mc}A|\psi|^2,$$

$\vec{A}$ is the vector potential of the magnetic field of the current, and L is the normalization length of the sample. We consider a cylindrical sample with the z axis along the sample axis. We can then assume that the only nonvanishing component of $\vec{A}$ is $A_z = -\pi j r^2/c$, where j is the total current density and r is the distance from the axis. We note that the magnetic field of the current is taken into account exactly, while the electric field E is assumed to be weak. The latter is justified because the conductivity of a sufficiently pure normal metal is high.

We expand the parameter ψ in terms of the eigenfunctions of the operator

$$\hat{L} = -[\nabla - (2ieA/c)]^2.$$

The only functions that matter in this expansion are those whose eigenvalues λ are close to the minimal eigenvalue. Such eigenfunctions have the form $e^{ikz}f_k(r)$, where the $f_k(r)$ can be chosen to be real and normalized by the condition

$$\int_0^\infty 2\pi r\,dr\,f_k^2 = 1.$$

The upper limit in the normalization integral is equal to infinity because, as we shall show, f_k differs from zero only when $r \lesssim \xi$ (ξ is the coherence length), and the sample radius is $R \gg \xi$.

The ordering parameter satisfies the temporal equation [5]

$$\frac{\partial\psi}{\partial t} = \nu(1 - \xi^2\hat{L})\psi,\quad \nu = 8(T_c - T)/\pi,$$

whence $\dot{a}_k = -\nu\sigma(k)a_k$, where $\sigma(k) = \xi^2\lambda(k) - 1$ and a_k are the coefficients of expansion in the eigenfunctions

$$\psi = \sum e^{ikz} f_k(r) a_k.$$

The quantity $\bar{j}_z$ in (1) is expressed in terms of a_k in the following manner:

$$\bar{i}_z = \frac{2e}{m} L \sum_k \Lambda(k) \,|\, a_k |^2 , \tag{2}$$

where

$$\Lambda(k) = k - (2e/c) \int_0^\infty 2\pi r\, dr\, A_z f_k^2$$

is the mean value of the operator $\partial\hat{L}/\partial$ multiplied by 1/2. Since the indicated mean value equals, as is well known, the derivative of the corresponding eigenvalue, $d\lambda/dk$, we have $\Lambda = (1/2)d\lambda/dk = (1/2\xi^2)d\sigma dk$. Using the foregoing temporal equation for a_k and formula (2), we can easily calculate the integral

$$\int_0^\infty dt\, e^{i\omega t} < \bar{i}_z(0)\, \bar{i}_z(t) > = -\frac{8e^2}{m^2} L^2 \sum_{kk'} \frac{\nu\sigma(k')\Lambda(k)\Lambda(k')}{i\omega - 2\nu\sigma(k)} \times \tag{3}$$

$$\times < |a_k|^2 \, |a_{k'}|^2 > .$$

The averaging over the fluctuations is carried out with the aid of the well known expression for the free energy

$$F = (2m\xi^2)^{-1} \int dV \psi^* (\xi^2 \hat{L} - 1)\psi = \frac{L}{2m\,\xi^2} \sum_k \sigma(k) \,|\, a_k |^2 .$$

We have

$$< |a_k|^2 \, |a_{k'}|^2 > = < |a_k|^2 > < |a_{k'}|^2 > + \left(\frac{2mT\,\xi^2}{L}\right)^2 \frac{\delta_{kk'}}{\sigma^2(k)} . \tag{4}$$

if we substitute in (3) the first term in the first part of the last equation, then we get zero, since the mean value of the product of two currents breaks up in this case into a product of two mean values. Only the second term of (4) makes a nonvanishing contribution. After simple transformations, we get from (1)

$$Q = \frac{2e^2 T}{L\nu} \sum_k \sigma^{-3}(k) \left(\frac{d\sigma}{dk}\right)^2 . \tag{5}$$

Since an exact calculation of the function $\sigma(k)$ is difficult, we shall use a variational principle, using as the trial function an expression of the form $f = (2\alpha/\pi)^{1/2}\exp(-\alpha r^2)$. The eigenvalue $\lambda(k)$ is minimal at $k = k_0 = -(\pi e j/c^2)^{1/3}$ and $\alpha = (\pi e j/c^2)^{2/3}$. We get for $\sigma(k)$ the expansion

$$\sigma(k) = 2(j - j_c)/3j_c + (3\xi^2/4)(k - k_0)^2 ,$$

where $j_c = (c^2/\pi e)(3\xi^2)^{-3/2}$ and it is assumed that $j - j_c \ll j_c$. Substituting this in (5), we obtain for the fluctuation correction to the effective conductivity

$$\sigma'_{\text{eff}} = \frac{Q}{\pi R^2} = 0{,}1\; \frac{e^2 \xi}{R^2} \; \frac{T_c}{T_c - T} \left(\frac{J_{c2}}{J - J_{c2}}\right)^{3/2} , \tag{6}$$

where $J_{c2} = \pi R^2 j_c \approx 0.19(c^2R^2/e\xi^3)$ is the sought second critical current.

From the point of view of the value of the fluctuation conductivity, the case of a hollow cylindrical sample is more convenient. In the case of a solid cylinder, a contribution to the fluctuation conductivity is made only by the region near the cylinder axis with a cross section area on the order of ξ^2. On the other hand, in the case of a hollow cylinder the corresponding region is located along the entire internal surface and its area is of the order of $R_1\xi \gg \xi^2$ (R_1 is the inside radius of the cylinder). Calculations similar to those presented above lead to the following expression for the fluctuation conductivity of a hollow cylinder (R_2 is the outer radius)

$$\sigma'_{\text{eff}} = 0{,}17\,\frac{e^2R_1}{R_2^2 - R_1^2}\;\frac{J_{c2}}{J - J_{c2}}\;\frac{T_c}{T_c - T}, \tag{7}$$

where $J_{c2} \approx 0.27[c^2(R_2^2 - R_1^2)/e\xi^3]$ is the second critical current for a hollow cylinder.

The ratio of the second critical current J_{c2} to the Silsbee current (J_{c1}) can be written in the case of a solid cylinder in the form

$$J_{c2}/J_{c1} = 1{,}1\,\kappa\,\frac{R}{\xi}, \tag{8}$$

where κ is the parameter of the Ginzburg-Landau theory. For a hollow cylinder, formula (8) also holds, provided the substitution $R = 1.4(R_2^2 - R_1^2)/R_2$ is made.

If the temperature approaches T_c, the length ξ increases. It is seen from (8) that at temperatures T such that $R/\xi(T) < 0.9\,\kappa$, the ratio J_{c2}/J_{c1} becomes smaller than unity. Since $\kappa \ll 1$ in pure superconductors, the sample is bulky in such a case ($R \gg \xi(T)$).

To understand the course of the process of superconductivity destruction with increasing current under the condition $J_{c2}/J_{c1} < 1$, it is sufficient to note that when $J < J_{c1}$ the magnetic field is smaller than H_c throughout the interior of the sample, and the superconducting state is therefore stable. Thus, if $J_{c2} < J_{c1}$, then the destruction of superconductivity occurs at the Silsbee point as before, but the sample goes over in this case directly into the purely normal state, and there is no region of intermediate state.

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