BEHAVIOR OF CROSS SECTIONS NEAR THE THRESHOLD OF REACTIONS WITH GIANT-RESONANCE FORMATION

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The energy dependence of the cross sections near the threshold of any inelastic process reveals root singularities ("cusps") [1]. For nuclear reactions, a cusp extends over several dozen keV.

We investigate in this paper the behavior of the cross sections of reactions in an energy interval in which there exists a set of thresholds of excitation thresholds of unstable particle states j (j = 0, 1, ..., N) having common decay channels. The distance between thresholds can be much smaller than the size of the cusp.

Let us consider the case when the width of one of the levels (with index 0) satisfies the inequality

$$\Gamma_0 \gg \Gamma_i \qquad (i = 1, 2, \ldots, N). \tag{1}$$

We assume henceforth that the states i are isolated from one another, i.e., $\Gamma_i/D \ll 1$, where D is the distance between neighboring levels. As shown in [2], the condition (1) leads to the appearance of a fine structure of the giant resonance in the cross section of the reactions that proceed via the channel (b + c) (see the figure).

The sought reaction amplitude will be determined by a sum of diagrams, one of which is shown in the figure:[1)]

$$M = -\left[\frac{2m_0 M_I^0 M_{II}^0}{p_Q^2 - 2m_0 E_0 - i m_0 \Gamma_0} + \sum_{i=1}^{N} \frac{2m_i M_I^i M_{II}^i}{p_i^2 - 2m_i E_i - i m_i \Gamma_i}\right]. \tag{2}$$

The total cross section of the reaction, determined by the amplitude M (Eq. (2)), is conveniently written in the form

$$\sigma_r = F \sum_{\ell, i=0}^{N} I_{\ell i} \qquad I_{\ell i} = r_\ell r_i e^{i(\phi_\ell - \phi_i)} \int_0^\infty \frac{p_a \, dp_a^2}{(p_a^2 - \lambda_\ell)(p_a^2 - \lambda_i^*)}. \tag{3}$$

We have introduced here the notation

$$2m_{ia}(E_{\text{kin}} - Q_i) + i m_{ia} \Gamma_i = \lambda_i; \quad 2m_{ia} M_I^i M_{II}^i = r_i e^{i\phi_i} \tag{4}$$

E_{kin} is the kinetic energy of particles A and B in the c.m.s., m_{ja} is the reduced mass, F is a constant, and p_a is the particle momentum in the c.m.s. of the reaction. The calculations yield

1) See the review [3] concerning the nonrelativistic diagram technique.

$$I_{ll} = A_l \mathrm{Re}\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})}; \; A_l = 2\pi r_l^2/\Gamma_l\sqrt{2m_{la}}\,, \tag{5}$$

$$I_{il} + I_{ll} = [B_i^{(l)} \mathrm{Re}\sqrt{(E - Q_i + i\frac{\Gamma_i}{2})} - C_i^{(l)} \mathrm{Im}\sqrt{(E - Q_i + i\frac{\Gamma_i}{2})}] +$$

$$+ [B_l^{(l)} \mathrm{Re}\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})} - C_l^{(l)} \mathrm{Im}\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})}]. \tag{6}$$

The constants in (6) are determined as follows:

$$\sqrt{2m_{la}}(C_i^{(l)} + iB_i^{(l)}) = \sqrt{2m_{ai}}(-C_l^{(i)} + iB_l^{(i)}) = 4\pi r_i r_l \sqrt{m_{ai}\, m_{al}} \; \frac{\exp[i(\phi_i - \phi_l)]}{\lambda_i - \lambda_l^*}. \tag{7}$$

It is seen from (7) and in (3) it is necessary to take into account only the contribution of the integrals I_{jj} and I_{i0}. The remaining quantities I_{ij} $(i \neq j \neq 0)$ contain small quantities of the order of Γ_i/D.

The sought cross section of the reaction is written in the form

$$\sigma_r = F \sum_{l=0}^{N} [D_l \mathrm{Re}\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})} - C_l \mathrm{Im}\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})}], \tag{8}$$

where

$$D_l = A_l + B_l^{(0)}; \quad B_0 = \sum_l B_0^{(l)}; \quad C_0 = \sum_l C_0^{(l)}.$$

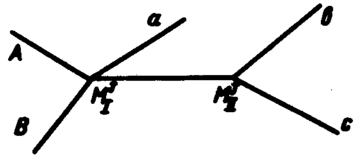
Following [1], we represent the analytic expression for the S-matrix element corresponding to elastic scattering in the form

$$S_0 = e^{2i\delta_0} - e^{2i\phi_0}\frac{k_\Pi^2}{2\pi} F \sum_{l=0}^{N} (D_l + iC_l)\sqrt{(E - Q_l + i\frac{\Gamma_l}{2})}, \tag{9}$$

where ϕ_0 is a real phase. In the case when there are no inelastic channels, the phase ϕ_0 coincides with the elastic-scattering phase δ_0. This yields elastic-scattering cross section in this region

$$\frac{d\sigma}{d_0} = |f_\Pi|^2 - |f_\Pi|\frac{k_\Pi}{2\pi} F \sum_{l=0}^{N} R_l\sqrt{|E - Q_l|} \times$$

$$\times \begin{cases} \sin(2\phi_0 - \alpha + \gamma_l) & \text{for} \quad E - Q_l >> \dfrac{\Gamma_l}{2} \\ \cos(2\phi_0 - \alpha + \gamma_l) & \text{for} \quad E - Q_l << -\dfrac{\Gamma_l}{2} \end{cases}. \tag{10}$$



The singularities near each threshold Q_j should be considered in the region $|E - Q_j| \gg \Gamma_j/2$. In (10) we have introduced the following symbols: $f_t = |f_t|e^{id}$ is the elastic-scattering amplitude, k_p is the wave number in the c.m.s. of the channel (A + B), $R_j = (D_j^2 + C_j^2)^{1/2}$, $\gamma_j = \tan^{-1} C_j/D_j$ is the phase obtained as a result of the overlap of the broad and narrow poles in formula (2). It follows from (10) that the elastic-scattering cross section is determined by the sum of the independent cusps situated at the points Q_j ($j = 0, 1, \ldots, N$). If the width Γ_0 is comparable with the extent of the cusp, then the cusp corresponding to the threshold Q_0 will be smeared out and will not appear in formula (10). However, the presence of a broad pole in (2) leads to a change in the type of the remaining cusps near the point Q_0, owing to the additional phases γ_j. These phases reach a maximum value for cusps located in the vicinity of the point Q_0, and vanish when $|Q_j - Q_0| \gg \Gamma_0/2$ (see formula (7)).

Thus, a study of the cross sections of the reactions at the production threshold of the giant resonance can answer the question whether the giant resonance is due to the presence of a broad pole or some other causes. In the latter case, the phases γ_j will not appear, and the change of the type of the cusp will be determined by the region of appreciable change of the elastic-scattering phases.

This question can be investigated experimentally by studying, say pp scattering from medium nuclei in the region of the direct pn reaction with analog-resonance production [4].

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