CRITICAL TEMPERATURE OF SUPERCONDUCTING TRANSITION IN NON-IDEAL CRYSTALS

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It is indicated in a number of recent experimental papers [1 - 4] that an appreciable change is apparently produced in the critical temperature of a superconductor by structural changes in the crystal. A consistent calculation of the superconducting characteristics of disordered crystals, using models with electron-phonon interaction, is made very difficult by the fact that the very concept of a phonon in a non-ideal crystal is not well defined [5, 6].

It has been shown, however [5 - 7], that the electronic properties of simple metals in-

cluding the kinetic characteristics, can be described in the language of the ion-density correlation functions $S(q, \omega)$, first introduced by Van Hove in the calculation of the scattering of slow neutrons in crystals

$$S(q, \omega) = \frac{1}{N}\int_{-\infty}^{+\infty} dt \exp(i\omega t) < \sum_{n,n'} a_n(q) a_{n'}(q) \exp[iqR_n(t)] \times \times \exp[-iqR_{n'}(0)] >, \quad (1)$$

where $R_m(t) = R_n(0) + u_n(t)$, N is the number of ions in the crystal, and $a_n(q)$ is the amplitude of scattering of the electron by the n-th ion; this amplitude is expressed in the Born approximation in terms of the pseudopotential of the electron-ion interaction $v_n(q)$

$$a_n(q) = \frac{m}{2\pi} v_n(q),$$

where m is the electron mass. The effective attraction of the electrons as a result of their interaction with the ions can be written in second-order interaction theory in the form [7]

$$V_{int}(q, \omega) = (2\pi/m)^2 \int_{-\infty}^{+\infty} \frac{S'(q, \omega')\, d\omega'}{\omega - \omega'}, \quad (2)$$

$$S'(q, \omega) = S(q \cdot \omega) - 2\pi\delta(\omega) \frac{1}{N} \left| \sum_n a_n \exp(iqR_n) \right|^2.$$

Calculation of the critical temperature for superconductors with weak coupling, where damping effects can be neglected, leads to the following result

$$T_c = \omega_0 e^{-1/\lambda}, \quad (3)$$

where ω_0 is the maximum frequency of ion oscillations,

$$\lambda = \frac{N(0)(2\pi)^2}{m^2} \int_0^{q_0} \frac{q\, dq}{2k_F^2} \int \frac{S'(q, \omega)}{\omega} d\omega, \quad (4)$$

N(0) is the density of states of the electrons on the Fermi surface, and k_F is the Fermi momentum of the electron. In deriving (3), we have neglected the presence of a direct Coulomb repulsion, and also the influence of the anisotropy. Allowance for all these effects does not change qualitatively the subsequently obtained results.

Formulas (3) and (4) allow us to connect the change of the critical temperature of a defect-containing superconductor with the quantities directly observed in high-temperature experiments, particularly with the change of the temperature-dependent part of the resistance of a metal. Indeed, the resistance of a metal at high temperatures can be written in the form [5, 6]

$$\rho(T)\big|_{T, \theta_D} = \eta T \int_0^{q_0} q^3 dq \int_0^{\infty} \frac{S'(q, \omega) d\omega}{\omega}. \quad (5)$$

Comparison of formulas (4) and (5) shows that the change of the coupling constant λ can be expressed in the form

$$\frac{\delta\lambda}{\lambda_0} \approx \frac{\delta\rho(T)}{\rho_0(T)}\Big|_{T>\theta_D}, \tag{6}$$

and consequently, the change of the critical temperature is

$$\frac{\Delta T}{T_c^0} \approx \ln\frac{\omega_0}{T_c^0}\frac{\delta\rho}{\rho}. \tag{7}$$

Calculations show that in the presence of weak disorder in the metal we have

$$\frac{\delta\rho}{\rho} \approx \frac{\overline{\delta R^2}}{R^2}, \tag{8}$$

where R is the mean distance between atoms and δR^2 is the mean-square deviation of the center of gravity of the atoms from their position in an ideal lattice.

The results are in qualitatively good agreement with the experimentally-observed [1 - 4] increase of the critical temperature in amorphous films, and the increase, as follows from (7), is most noticeable in superconductors with large values of ω_0/T_c. Notice should be taken of the correlation observed in these experiments between the change of the critical temperature and the resistance, indicating that the connection between them is linear.

A quantitative comparison with the experimental data is made difficult by the poor knowledge of the true structure of these films, and also by the influence of the sample boundaries, which apparently leads to an appreciable dependence of T_c on the thickness.

Formula (7) allows us also to indicate the reasons why the impurities have little influence on the change of T_c. Although the presence of impurities can lead to an appreciable change of the lattice dynamics, and to the occurrence of local and quasilocal frequencies, thereby strongly affecting a large number of physical phenomena, these effects are due essentially to the change of the impurity mass. As shown in [5], at high temperatures $\Delta\rho/\rho$ does not depend on the impurity mass, and consequently the entire change of T_c is connected with the change of either the amplitude of the scattering by the impurity, or with the change of the force constants, which is a small quantity in simple metals.

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