DRAGGING OF ELECTRONS BY AN ELECTROMAGNETIC WAVE IN A HOMOGENEOUS MAGNETIC FIELD

V. P. Oleinik
Semiconductor Institute, Ukrainian Academy of Sciences
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Kibble [1, 2] has shown that an electromagnetic behaves relative to an electron of energy ε like a medium with a refractive index

$$n = [1 - \frac{e^2\bar{E}^2}{\omega^2(\epsilon^2 - m^2)}]^{1/2},$$

and therefore electrons, whose energy satisfies the condition[1)]

$$\epsilon^2 - m^2 < (e^2\bar{E}^2/\omega^2), \tag{1}$$

are reflected from the wave ($\bar{E}^2$ and ω are the time-averaged squared intensity of the electric component and the frequency of the wave). In this communication we call attention to the fact that in the presence of a homogeneous magnetic field with potential

$$A_x = -H_y \tag{2}$$

the electrons penetrate into the wave regardless of their energy, and when ϵ is lower than a certain critical value all the electrons that enter the wave are dragged by it and acquire a relativistic momentum in the wave propagation direction.

The problem of electron penetration in the wave can be solved by the usual methods employed to investigate the penetration of particles through a potential barrier (cf., e.g., [3]). If the electromagnetic wave is described by a potential

$$\mathbf{A} = \begin{cases} \mathbf{a}\cos\omega(t - z) & \text{for} \quad z > 0 \\ 0 & \text{for} \quad z < 0; \quad \mathbf{a} = (a_x, a_y, 0) = \text{const}, \end{cases} \tag{3}$$

then the condition for the continuity of the electron wave function and its derivative with respect to z at z = 0 leads to equations for the determination of the intensity of the electron fluxes reflected from the wave and penetrating into the wave; it also follows from this condition that the energies of the incident and penetrating electrons are equal.

In the field of the electromagnetic wave, the role of the energy ϵ of the stationary states of the electron is played by the quasienergy [4], so that the comparison should be made between the energy of the incident electron ϵ and the quasienergy ϵ' of the electron in the wave. The connection between the quasienergy ϵ' and the quasimomentum p_z of an electron in the fields of formulas (2) and (3) is given by the following formula from [5]

$$\epsilon'^2 - m^2 = p_z^2 + eH(2n+1) - \frac{e^2 a^2}{2}\,\frac{\omega^2(\epsilon' - p_z)^2}{(eH)^2 - \omega^2(\epsilon' - p_z)^2}, \tag{4}$$

where n = 0, 1, ..., n is the number of the Landau level of the electron in the wave (to simplify the derivations, we describe the interaction of the electron with the external field by a Klein-Gordon equation). Replacing ϵ' in formula by ϵ, we obtain the sought relation, which we shall investigate for an electromagnetic wave of frequency ω, in the optical region of the spectrum ($\omega \gg \omega_H = eH/\epsilon$), and for nonrelativistic electron energies ($\epsilon^2 - m^2 \ll m^2$). In this case, as can be readily verified, if the condition

1) We use a system of units in which $c = \hbar = 1$, and also the relativistic reference point of the energy.

$$\varepsilon^2 - m^2 < \frac{e^2 a^2}{2} \frac{\omega^2}{\omega^2 - \omega_H^2} \approx \frac{e^2 a^2}{2} \qquad (5)$$

is satisfied, only electrons with very large quasimomenta $\bar{p}_z$ can propagate in the wave, and at fixed values of n the electrons can have only two values of p_z (the other two roots of (4) are complex):

$$p_{z1,2} = \varepsilon(1 \pm \Delta); \; \Delta \approx \frac{\omega_H}{\omega}\left[1 + \frac{e^2 a^2}{2[m^2 + eH(2n+1)]}\right]^{-1/2}. \qquad (6)$$

When H = 0 and the electron energies correspond to the inequality (5), Eq. (4) has only complex roots, i.e., in this case the electrons are reflected from the wave, as they should.

When the inequality opposite to (5) is satisfied, there appears in the electromagnetic wave, besides the fast electrons, also an additional group of electrons (slow electrons) for which the maximum value of the longitudinal quasimomentum is

$$p_z^* = \sqrt{\varepsilon^2 - m^2 - \frac{e^2 a^2}{2}}.$$

For modern laser sources we have $e^2a^2 \ll m^2$, so that for this group of electrons $p_z^* \ll \varepsilon$. The two groups of electrons are thus separated by a broad forbidden band for the quasimomenta p_z. In this limiting case, the characteristic features of the fast and slow electrons in the field of the electromagnetic wave consist of the fact that the fast electrons appear in the form of two monochromatic streams with a relativistic quasimomentum directed along the wave vector of the wave, whereas the slow electrons have momenta that lie in the interval $0 < p_z < p_z^*$.

In the phenomenon considered here, the electromagnetic wave behaves like a stream of liquid, dragging with it the particles (electrons) that fall into it. At sufficiently low energy ε, all the particles entering the liquid are dragged, and only partial dragging of the particles take place if the energy is larger. From the point of view of this analog, it is natural that the "dragged" electrons have relativistic momenta in the direction of motion of the photon beam.

The problem of determining the coefficient of penetration of the electron into the wave, D, reduces in the general case to a solution of an infinite system of linear algebraic equations. It simplifies greatly for an electron energy satisfying the condition (5), and for optical frequencies ($\omega \gg \omega_H$), for in that case the electrons propagating in the wave have quasimomenta p_{z1} and p_{z2} that depend weakly on n.

Neglecting this dependence and the difference between p_{z1} and p_{z2}, we can eliminate the wave function of the electron in the wave from the system of equations obtained when the wave functions are made continuous on the boundary z = 0. This yields for the penetration coefficient the formula

$$D = 4\frac{\varepsilon \bar{p}_z}{(\varepsilon + \bar{p}_z)^2}, \qquad (7)$$

where $\bar{p}_z = [\varepsilon^2 - m^2 - eH(2n + 1)]^{1/2}$ and n is the number of the Landau level of the incident electron. In the most favorable case, described by formula (7), $\bar{p}_z^{\,2} \sim e^2a^2/2$, and therefore $D \sim 4[e^2a^2/2m^2]^{1/2}$. In focused radiation from a modern laser source, e^2a^2/m^2 can reach values on the order of 10^{-6}, and then $D \sim 3 \times 10^{-3}$.

The effect proposed here can be observed experimentally with the aid of an electron gun whose cathode is placed in the field of a light beam. According to (7), the penetration coefficient will then be larger for the electrons emitted from the cathode at a smaller angle θ to the direction of the wave vector. In setting up the experiment it must be borne in mind that the homogeneous magnetic field should be sufficiently strong. In fact, it has been assumed here that the wave occupies the entire half-space $z > 0$. In a real experiment, on the other hand, the high-intensity light beam has a finite radius R. Therefore, our results remain valid only when the radius of the electron orbit in the magnetic field, $r = p_\perp/eH$, satisfies the condition $r \ll R$ ($p_\perp$ is the projection of the momentum of the incident electron on the xy plane). When $\theta \ll 1$ and $p \sim e|\vec{a}|$ (p is the total momentum of the incident electron) we obtain the following limitation on the magnetic field intensity ($p_\perp \simeq p$, c is the speed of light):

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