# DETERMINATION OF THE CHARGE OF A HEAVY NUCLEUS

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Much attention has been paid recently to Fowler's reports of detection of nuclei with charge $Z = 92 \pm 4$ in cosmic rays [1], and according to the latest data also with $Z = 92$, 104, and 108. Fowler has assumed that his determination of Z is accurate to $\sim 2\%$.

We present in this paper a refinement of the theoretical formulas used to determine the charges of relativistic heavy nuclei.

As is well known, in the case of electromagnetic interactions of nuclei with light and medium elements moving at very low velocities, the Born approximation is valid with sufficient accuracy. In this approximation, the density of the emulsion blackening, the ionization loss in the chamber with the rarefied gas, and the Cerenkov radiation loss are all proportional to $Z^2$. However, for nuclei whose charge satisfies the condition $Z/137\beta \sim 1$, it is necessary to take into account the next higher terms of the perturbation-theory series. Here $\beta$ is the velocity of the incoming nucleus.

The estimates that follow show that the calculated corrections exceed the indicated experimental errors of Fowler's results already in the second Born approximation.

The need for taking into account the higher-orders of perturbation theory was already encountered earlier in an analysis [2, 3] of the experimental data on the electromagnetic structure of heavy nuclei (Hofstadter's accelerator experiments). In the second order of perturbation theory, the cross section for the scattering of electrons by the Coulomb field of a heavy nucleus was obtained in analytic form [2, 4].

It is obvious that in the case of δ-electron production by a heavy nucleus we have the inverse kinematic problem. If we use the results of Feshbach [2] and Dalitz [4], then a simple kinematic recalculation from the c.m.s. of the colliding particles to the rest system of the emulsion nuclei, followed by elementary integration of the differential cross section, yields a formula for the number of δ-electrons per centimeter of heavy-particle track, accurate to terms of order $(Za)^3$:

$$N_\delta = 2\pi N r_0^2 mc^2 (Z^2/\beta^2)\{\frac{1}{T_{min}} - \frac{1}{T_{max}} - \frac{1-\beta^2}{2m}\ln\frac{T_{max}}{T_{min}} + \pi Z a[2\sqrt{1-\beta^2}(\frac{1}{\sqrt{2mT_{min}}} - \frac{1}{\sqrt{2mT_{max}}}) - \frac{1-\beta^2}{2m\beta}\ln\frac{T_{max}}{T_{min}}]\}, \quad (1)$$

where N is the number of electrons per $cm^3$ of emulsion, m mass, r the classical radius of the electron, and $T_{min(max)}$ the minimal (maximal) registered δ-electron energy.

We assume that $T_{min}$ = 50 keV and $T_{max}$ = 150 keV. We note that the range of 50-keV δ electrons is $R_{min} \simeq 10\ \mu$ for an emulsion with density $\rho = 4$ g/cm$^3$. The range $R_{min}$ corresponds to that minimum distance from the "core" of the heavy-particle tracks, at which the photometry of the blackening produced by the particle is started (∿ 10 μ in Fowler's experiments).

Since the photographic density of the silver grains is proportional to the ionization or to the number of δ electrons [5], the correction for the second approximation has the same form whether one counts the number of δ electrons or grains in the emulsion, or whether one uses the photometric method.

The correction as a whole is determined by the expression in the brackets of (1) and equals

$$\Delta = \frac{N_\delta - N_\delta^1}{N_\delta}, \quad (2)$$

where $N_\delta^1$ is the number of δ electrons in the Born approximation.

The value of Δ does not depend on the common factor $(\beta^2)^{-1}$, the influence of which is appreciable if the particle velocity is inaccurately determined.

In Fowler's experiments [1] no account was taken of the second perturbation-theory approximation, since the unknown charge was determined by comparing the photographic densities produced by a relativisitc iron nucleus ($D_{Fe}$) and by the unknown heavy particle ($D_x$), using the formula

$$Z_x = 26\sqrt{\frac{D_x}{D_{Fe}}}, \quad (3)$$

It is assumed here that the particles have an equally large velocity ($\beta \sim 1$), so that the photographic density does not depend on the velocity and varies in proportion to $\sim Z^2$.

For the iron nucleus ($Z = 26$), the introduced correction is approximately smaller than for the Te nucleus ($Z = 52$) (see the table), i.e., the first Born approximation actually suffices for iron. For the remaining nuclei, the charge should be reduced in accordance with the formula

$$Z = Z^{(1)}(1 - \Delta/2), \qquad (4)$$

where $Z^{(1)}$ is the charge determined from the formulas of the Born approximation.

The correction is particularly large for high geomagnetic lattitudes, where the smallness of the velocity $\beta$, which enters into the Coulomb parameter $Z\alpha/\beta$, comes into play. In this case, however, it is necessary to take into account all the succeeding terms of the perturbation-theory series, or else to use numerical calculations with the aid of a phase-shift analysis [3].

The numerical results are listed in the table ($|\Delta Z| = |Z^{(1)} - Z|$).

| $\beta$ | Te (Z = 52) | | U (Z = 92) | | Z = 114 | |
|---|---|---|---|---|---|---|
| | Δ % | \|ΔZ\| | Δ % | \|ΔZ\| | Δ % | \|ΔZ\| |
| 0,85 | 12,5 | >3 | 20,1 | >9 | 23,3 | >13 |
| 0,92 | 10,4 | >2 | 17 | ~7 | 20 | ~11 |
| 0,95 | 8,7 | >2 | 14,4 | >6 | 17,3 | ~10 |
| 0,99 | 4,2 | >1 | 7,2 | >3 | 8,8 | ~5 |

It is also easy to estimate the contribution of the second Born approximation to the ionization losses when a heavy particle passes through a chamber with a rarefied gas. In this approximation, the calculations were made without allowance for the bound states of the electrons in the atoms of the target gas, and the numerical results correspond approximately to the numbers indicated in the table. It seems to us that in the Cerenkov-counter method it is also necessary to take into account the emission of two photons by the heavy nucleus (second Born approximation).

In conclusion, the author thanks I. L. Rozenthal' and G. N. Flerov for useful discussions.

[1] P. H. Fowler, R. A. Adams, V. G. Cowen, and I. M. Kidd, Proc. Roy. Soc. A301, 39 (1967).
[2] W. McKinley and H. Feshbach, Phys. Rev. 74, 1759 (1948).
[3] D. R. Yennie, D. G. Ravenhall, and R. N. Wilson, Phys. Rev. 95, 500 (1954).
[4] R. H. Dalitz, Proc. Roy. Soc. A206, 509 (1951).
[5] C. F. Powell et al., Study of Elementary Particles by the Photographic Method, Pergamon, (1959).

# E R R A T A

In the article by I. M. Arf'ev and V. V. Morozov, Vol. 9, No. 8, "stimulated thermal scattering" on lines 19-20 of p. 269 should read "stimulated temperature scattering.'

In the article by V. B. Semikoz, Vol. 9, No. 9, "approximately smaller than" on line 3 of p. 326 should read "approximately smaller by a factor of two than."