CONCERNING MASS SPLITTING IN ISOMULTIPLETS OF PSEUDOSCALAR MESONS

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In this paper we calculate (to the order of e^2) the radiative corrections to the masses of the pseudoscalar mesons in the Lagrangian theory with transverse vector fields [1] (see the recent discussion in [2]):

$$L(x) = -\frac{1}{4} V^i_{\mu\nu} V^i_{\mu\nu} - \frac{1}{2} V^i_\mu m_V^2 V^i_\mu + L_m(\Psi, D_\mu \Psi), \tag{1}$$

$$V^i_{\mu\nu} \equiv \partial_\mu V^i_\nu - \partial_\nu V^i_\mu + g C_{ijk} V^j_\mu V^k_\nu, \quad D_\mu \Psi \equiv \partial_\mu \Psi + T^i V^i_\mu \Psi,$$

where C_{ijk} are the structure constants of the compact semi-simple Lee algebra, and the matrices T^i specify the representation of this algebra in the space of the fields Ψ. The symmetry level of interest to us in (1) corresponds to invariance of L(x) relative to the group $SU(2)$ $U(1)_B$ $U(1)_Y$ (the direct product of the iospin, baryon charge, and hypercharge groups). The source of this invariance, according to [1], is the triplet of vector fields V^1, V^2, V^3 and the pair of singlets V^4 and V^5 ($C_{ijk} = \epsilon_{ijk}$; i, j, k = 1, 2, 3 and $C_{ijk} = 0$ for other values of the indices i, j, k) and according to the universally accepted identification, due to Sakurai [3] (and with allowance for the ω-ϕ mixing), these fields are $V^i_\mu \equiv (\rho_\mu, \omega_\mu, \phi_\mu)$, i = 1, 2, 3, 4, 5.

We now specify L_m in detail. We take into account all the intermediate single-particle states that make contributions in the Kroll-Lee-Zumbino electrodynamics to the electromagnetic self-energy of the pseudoscalar mesons P. In the minimal Lagrangian theory (theory with dimensionless coupling constants and with constants having the dimension of mass), these are the states with $J^P = 0^-, 1^+, 2^-$. Nothing is known at present concerning 2^- resonances.

Therefore, retaining in L_m only the part pertaining to the 0^- and 1^+ fields, we get

$$L_m(x) = a[-\frac{1}{2} D_\mu P D_\mu P - \frac{1}{2} P \mu^2 P] - \frac{1}{4} A_{\mu\nu} A_{\mu\nu} - \frac{1}{2} A_\mu m^2 A_\mu + \tag{2}$$

$$+ f A_\mu D_\mu P,$$

where $A_{\mu\nu} \equiv D_\mu A_\nu - D_\nu A_\mu$, P and A_μ describe the isomultiplets of the 0^- and 1^+ mesons, respectively, and $\alpha \neq 1$ characterizes the P-A mixing. Diagonalization of L_m with respect to A_μ and $\partial_\mu P$

$$A_\mu = a_\mu + \frac{f}{m^2} \partial_\mu P, \quad a = 1 + \frac{f^2}{m^2}. \tag{3}$$

leads to new fields a_μ, with which the 1^+ mesons will henceforth be identified. The 3- and 4-vertices of interest are of the form

$$L_{VPP} = V_\mu^l P T^l \partial_\mu P, \tag{4}$$

$$L_{AVP} = \frac{f}{m^2} V_\mu^l [(\partial_\mu a_\nu - \partial_\nu a_\mu) T^l \partial_\nu P + m^2 a_\mu T^l P], \tag{5}$$

$$L_{VVPP} = \frac{1}{2} V_\mu^l V_\nu^j [\delta_{\mu\nu} \partial_\lambda P (T^l T^j) \partial_\lambda P - \partial_\nu P (T^j T^l) \partial_\mu P - a \delta_{\mu\nu} P (T^l T^j) P] \tag{6}$$

in accordance with the types of diagrams possible in our theory. The low degrees of the internal 4-momenta on these diagrams guarantee a finite self-energy of the P mesons in the gauge-invariant Kroll-Lee-Zumino formulation. We note that the free Lagrangian of the 1^+ mesons is chosen in (2) in such a way that the charged componenets of the multiplets have unity magnetic moments μ (in units of $e/2m_A$). Because of this, no couplings of the type $\partial_\mu V_\nu A_\mu A_\nu$ arise in L_m, capable of leading to high degrees of the integration momenta in the vertices (4) and (5) after diagonalization of (3), and thus leading to diverging integrals in the self energy. However, this very interesting observation, made in [5], does not solve the problem as a whole. Thus, the electromagnetic self-energies of the V and A mesons remain infinite at all values of μ_V and μ_A.

We now proceed directly to a determination of the mass splittings.

1) We consider the isovector multiplet

$$P \equiv \vec{\pi}, \quad a \equiv A_1(1070), \quad T_{jk}^l V_\mu^l = g \epsilon_{ljk} \rho_\mu^l \quad (\frac{g^2}{4\pi} = 2, 3\ [4]). \tag{7}$$

Using (4) - (6), the form of the effective photon propagator in [4] ($D_\gamma(k^2) \to O(k^{-6})$ as k^2 $k^2 \to \infty$), and the experimental value 4.6 meV, of the $\pi^+ - \pi^0$ mass difference [6], we get $f_V \approx m_\rho$ and $\Gamma(A_1 \to \rho\pi) = 150$ meV. Another possible candidate (besides $\vec{A}_1$) is $\vec{B}(1220)$ with $J^{PG} = 1^{++}$. It is easy to see that it makes identical contributions to the π^+ and π^0 masses.

2) Let us consider in greater detail the case of the $K^+ - K^0$ mass difference (isospinor multiplet):

$$P \equiv K, \quad a \equiv K_A(1320), \; T^i V_\mu^i = g \frac{\tau^i}{2} \rho_\mu^i + (g_\omega \omega_\mu + g_\phi \phi_\mu) I, \tag{8}$$

where τ^i are Pauli matrices and I a unit matrix. In the current-mixing model [4] we have[1)]

$$g_\omega = g_Y \frac{\sin\theta_B}{\cos(\theta_Y - \theta_B)}, \; g_\phi = g_Y \frac{\cos\theta_B}{\cos(\theta_Y - \theta_B)} (\theta_B = 21°, \quad \theta_Y = 33°, \; \frac{g_Y^2}{4\pi} = 1,5) \tag{9}$$

and for the mass difference (in the Landau gauge)

$$\delta\mu^2_{K^+} - \delta\mu^2_{K^0} = \frac{ie^2}{(2\pi)^4} \int d^4k \left[\frac{R_K}{(k-p)^2 - \mu^2} + \frac{f_s^2 R_{K_A}}{(k-p)^2 - m^2} \right] F(k^2), \tag{10}$$

where

$$k^2 F(k^2) = (1-\beta) n_\rho n_\omega + \beta n_\rho n_\phi, \quad \beta \equiv \frac{\cos\theta_Y \cos\theta_B}{\cos(\theta_Y - \theta_B)},$$

$$n_i = \frac{m_i^2}{k^2 - m_i^2}, \quad i = \rho, \omega, \phi,$$

and R_K and $f_s^2 R_{K_A}$ are the contributions of vertices [4] and [5], respectively (vertex [6] makes no contribution to the mass difference)

$$1/_4 R_K = -\mu^2 + \frac{(kp)^2}{k^2},$$

$$R_{K_A} = 3 - 5\gamma + 2\gamma^2 + (6 - 4\gamma) \frac{kp}{m^2} + 3 \frac{(kp)^2}{m^4} - (1-\gamma) \frac{(kp)^3}{k^2 m^2} -$$

$$- 2 \frac{(kp)^3}{k^2 m^4}, \quad \gamma = \frac{\mu^2}{m^2}.$$

A value of $\delta\mu$ that agrees with the experimental result of -2.95 ± 0.21 of Hill et al. [6] is obtained from $f_s = m \equiv m_{K_A}$. From this we get for the $K_A(1320)$ partial widths $\Gamma(K_A \to \rho K) \approx 60$ meV and $\Gamma(K_A \to \omega K) \approx 6$ meV. Possible candidates (besides $K_A(1320)$) are also the poorly identified 1^+ quartets $K_A(1240)$, $K_A(1280)$, and possibly also a number of others in the interval 1200 - 1350 meV. Allowance for their contribution to $\delta\mu$ can greatly decrease the reduced widths. Thus, allowance for $K_A(1240)$ under the assumption of "universality" of the constant f_s for all

[1)] In addition, we actually use in this section the SU(3) forbiddenness of the coupling of the unitary-singlet parts in ω and ϕ with the P - a system (under the condition that the octet C-parity of the 1^+ mesons equals +1).

K_A quartets (which is quite likely) leads to $\Gamma(K_A(1320) \to \rho K) \approx 45$ meV and $\Gamma(K_A \to \omega) \approx 4$ meV $(f_s \approx \sqrt{2m})^{2)}$. Consequently exact measurement of one of the partial $K_A(1320)$ widths, say $\Gamma(K_A \to \rho K)$, might cast light on the existence of other K_A excitations. Experimental observation of a width 25 - 50 meV (the exact value of the width depends on the value of the resonance mass in the 1200 - 1350 meV resonance) would serve as evidence of the presence of one more K_A quartet (besides $K_A(1420)$), and observation of a width much smaller than 25 meV would indicate the presence of at least two other K_A quartets. We note that our recommendations remain in force also when 1^- fields are also included in L_m (of course, now already within the framework of the nominimal theory). The corresponding contributions to the mass splittings were estimated by Sokolov on the basis of SU(3) symmetry [7]. They have a positive sign and amount to not more than 10 - 15% of the experimental values of the mass differences. Thus, the considered approach affords a definite possibility of using rather exact measurements of the isomultiplet mass differences in order to obtain information on the little-investigated 1^+ objects.

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E R R A T A

In the article by Dzh. L. Chkareuli, published in Vol. 9, No. 9, the mass terms in formulas (1) and (2) should enter with positive signs, and formula (3) should read:

$$A_\mu = a_\mu - \frac{f}{m^2} d_\mu P\,, \qquad a = 1 + \frac{f^2}{m^2}$$