USE OF MAGNETS WITH VERTICAL SYMMETRY PLANES IN RING ACCELERATORS

V. N. Melekhin
Institute of Physics Problems, USSR Academy of Sciences
Submitted 2 April 1969
ZhETF Pis. Red. 9, No. 9, 552-556 (5 May 1969)

The existing strong-focusing ring accelerators employ magnets with horizontal symmetry planes to effect alternating focusing of the particles in the vertical and radial directions. Obviously this is not the only possible method, and the particles can be focused in any other direction, using magnetic fields of appropriate configuration. Let us point out certain features of a system that focuses the particles in mutually perpendicular directions ξ and η axes in Fig. 1) inclined 45° to the radial and vertical directions. Such a motion is real-

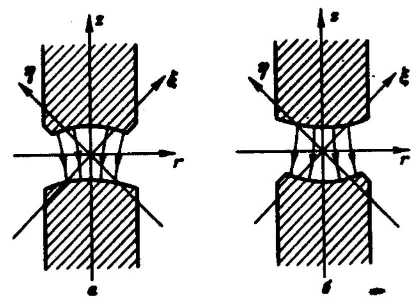

Fig. 1. Magnets with vertical symmetry planes.

[2] It is of interest to note that this value of f_s is closer to the value predicted by SU(3) symmetry: $f_v = f_s$.

ized, for example[1)], when magnets are used with a vertical symmetry plane[2)] as shown schematically in Figs. 1a and 1b. The magnetic field has in this case the form

$$H_z = -H_0 + nz, \quad H_\rho = -n\rho, \tag{1}$$

where H_z and H_ρ are the vertical and radial components of the field, and z and ρ are the vertical and radial deviations from the equilibrium orbit. We note that in the case shown in Fig. 1a $n > 0$, and in the case of Fig. 1b $n < 0$, it follows from (1) that in the linear approximation the field increases or decreases in the vertical direction, and does not change radially.

Confining ourselves for simplicity to a system consisting of alternating magnets of the indicated type (Figs. 1a and 1b) without gaps between them, we write down the equations of the transverse motion in terms of the coordinates ξ and η:

$$\begin{aligned} \frac{d}{dt}(E\dot{\xi}) - E\omega^2 n\xi &= \frac{1}{\sqrt{2}}E\omega^2 R\frac{\Delta p}{p}, \\ \frac{d}{dt}(E\dot{\eta}) + E\omega^2 n\eta &= -\frac{1}{\sqrt{2}}E\omega^2 R\frac{\Delta p}{p}, \end{aligned} \tag{2}$$

where E, ω, and R are the equilibrium values of the energy, the revolution frequency, and the radius, $\Delta p/p$ is the deviation of the momentum from the equilibrium value, and the quantity n reverses sign on going to each successive magnet. Both equations coincide in form with the equation for the radial-phase oscillations in ordinary strong-focusing ring accelerators, and their solution has been thoroughtly investigated (cf., e.g., [1]) and describes alternating focusing. It follows therefore that in our system particles with equilibrium energy oscillate along the axes ξ and η in exactly the same manner as they oscillate vertically and radially (betatron oscillations) in the ordinary strong-focusing system.

If the particle energy differs from the equilibrium value, then the induced part of the solution of the inhomogeneous equations (2) contains, just as in the case of ordinary synchrotron oscillations [1], a constant term and an oscillating one with a period equal to the period of the magnetic system. From the form of Eqs. (2) it can be concluded that the constant terms in the solutions of the two equations are equal in magnitude and opposite in sign, and the oscillating terms coincide fully. It follows therefore, after converting to the coordinates r and z, that the closed trajectory of a non-equilibrium particle is shifted radially by a constant amount equal to the average radial shift in the ordinary system, and oscillates in a vertical direction. These oscillations give rise to a second-order force containing the non-equilibrium particles.

---

1)One possibility is to use a magnetic system with separated functions, in which the focusing elements are suitably oriented; however, we shall not discuss this system.

2)As pointed out to the author by A. A. Kolomenskii, similar magnets were proposed for use in the CERN 30-GeV proton synchrotron, but preference was given in that project to magnets with horizontal symmetry planes.

The main advantage of the described system, in our opinion, lies in the symmetry of the transverse motions: the betatron oscillations are executed along axes that are symmetrical with respect to the radial plane, which in cyclic accelerators is singled out by virtue of the circular character of the motion. This advantage becomes manifest, for example, when account is taken of the radiation reaction. As is well known, in strong-focusing electron synchrotrons for radiation forces give rise to undesirable radial betratron oscillations. At the same time, these forces damp out the vertical oscillations, and the sum of the vertical and radial decrements equals zero [1]. In our case the sum of the decrements is equal to zero, as before, but now, by virtue of the symmetry of the problem, they are equal to each other, and consequently each equals zero, i.e., no oscillations are built up. This conclusion is confirmed by direct calculations.

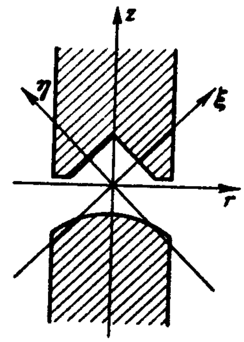

Fig. 2. Pole shape ensuring a large field gradient.

The symmetry of the system is also maifest in the unique influence of the nonlinearity of the field. In ordinary systems, the quadratic nonlinearity shifts the frequency of the betatron oscillations of particles having an energy different from the equilibrium value. In our system, as shown by calculation, there is no such frequency shift, and a difference-type of coupling between oscillations arises if the quadratic nonlinearity does not disturb the symmetry of the system relative to the vertical plane.

Let us note several additional distinguishing features of magnets with vertical symmetry planes. Using identical magnets with C-shaped construction, we can dispose these magnets on one side of the vacuum chamber, on the inner or outer radius. This simplifies the construction and operation of the accelerators, compared with the ordinary case, when the magnets must be located on both sides of the vacuum chamber. Magnets with polepieces shaped as shown in Fig. 2 can be used in those cases when it is necessary for some reason to increase the gradient of the magnetic field. Finally, by alternating magnets with vertical symmetry planes and ordinary magnets, it is possible to effect the so called spiral focusing, which has certain advantages.

In our opinion, systems with vertical symmetry planes are of undisputed interest and their properties should therefore be studied in greater detail and compared with existing systems. The absence of radiative buildup of betratron oscillations, which is characteristic of the considered system, uncovers new possibilities in electron synchrotrons for ultrahigh energies (for example, it permits an increase of the acceleration time) and in electron-positron storage rings.

[1] A. A. Kolomenskii and A. N. Lebedev, Teoriya tsiklicheskikh uskoritelei (Theory of Cyclic Accelerators), M., 1962.

[2] V. S. Zahkarov and M. S. Rabinovich, Zh. Tekh. Fiz. 34, 1986 (1964) [Sov. Phys.-Tech. Phys. 9, 1528 (1965)].

[3] The Theory and Design of an Alternating-gradient Proton Synchrotron (Report Presented at the Conference on the Alternating-gradient Proton Synchrotron, Geneva, 1953).