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EXPERIMENTAL DETERMINATION OF THE ANGULAR DEPENDENCE OF THE ELECTRON REFLECTION COEFFICIENT

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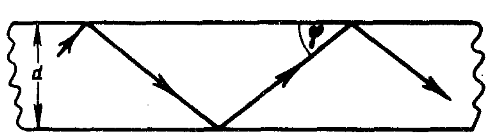
The dependence of the conductivity σ of filamentary whiskers and plates on the temperature T has been measured in a number of experiments (cf., e.g., [1 - 3]). The measurement results were interpreted on the basis of the assumed temperature dependence of the coefficient q of specular electron reflection from the surface of the metal. Yet it is difficult to expect a significant dependence of q on T at the low temperatures used in the cited studies.

We shall show that the results of these investigations can be naturally attributed to the well-known appreciable dependence of q on the angle ϕ of incidence of the electrons on the surface of the plate (see the figure) in the region of small ϕ, when the electron reflection is close to specular. Moreover - and this is particularly important - from the form of $\sigma(T)$ we can reconstruct the function $q(\phi)$ (see [4] on this subject; $q(\phi)$ was investigated theoretically in [5 - 7]).

To simplify the exposition, let us examine in greater detail the case of a plate that is thin in comparison with the electron mean free path ℓ. If the electrons moving at an angle ϕ were not to experience any collisions in the volume, they would be scattered in each collision with the surface with a probability $1 - q(\phi)$ and would continue their path without scattering with a probability $q(\phi)$. The path negotiated by them without scattering is (see the figure, d is the plate thickness)

$$\lambda(\phi) \equiv \frac{d}{\phi} + \frac{d}{\phi} q(\phi) + \frac{d}{\phi} q^2(\phi) + \ldots = \frac{d}{\phi(1 - q(\phi))}. \quad (1)$$



Collisions inside the volume limit the average path ℓ_{eff} traversed without scattering to a value on the order of ℓ, so that it can be assumed that ℓ_{eff} is of the order of $\lambda(\phi)$ if $\lambda(\phi) < \ell$, and $\ell_{eff} \sim \ell$ if

$\lambda(\phi) > \ell$. This means that

$$\ell_{eff}(\phi) \sim [(1/\ell) + (1/\lambda(\phi)]^{-1}. \tag{2}$$

The fraction of the electrons reflected in the angle interval from ϕ to $\phi + d\phi$ and acquiring energy from the field along the path $\ell_{eff}(\phi)$ is proportional to $d\phi$. Therefore the total current I, and hence the average electric conductivity $\sigma = I/Ed$, where E is the electric field intensity, are proportional to

$$\int_{-\pi/2}^{\pi/2} \ell_{eff}(\phi) d\phi.$$

For σ we have

$$(\sigma/\sigma_\infty) \approx (3/2)\xi \int_0^{\pi/2} (d\phi)/[\xi + \phi(1 - q(\phi))]^{-1} \tag{3}$$

When $\xi \ll 1$, the proportionality coefficient is obtained from the exact formula for the average electric conductivity of the plate, which is given in the case of isotropic dispersion by

$$\frac{\sigma}{\sigma_\infty} = \frac{3}{2\xi} \int_0^{\pi/2} d\phi \cos^3\phi \left[\xi - \left(1 - e^{-(\xi/\sin\phi)}\right) \frac{1 - q(\phi)}{1 - q(\phi)\, e^{-(\xi/\sin\phi)}} \sin\phi\right], \tag{4}$$

where σ_∞ is the specific electric conductivity of the specimen in bulk, and $\xi = d/\ell$.

The main contribution to (3) is made by the region of angles for which

$$\Phi(\phi) \equiv \phi(1 - q(\phi)) \sim \xi. \tag{5}$$

Therefore

$$\sigma/\sigma_\infty \sim \phi^*, \tag{6}$$

where ϕ^*, which has of course only an order-of-magnitude meaning, is determined from the relation[1)]

$$\Phi(\phi^*) \equiv \phi^*(1 - q(\phi^*)) = \xi. \tag{7}$$

The experimentally measured $\sigma(T)$ dependence (at a known $\ell(T)$ dependence) makes it possible to determine $\sigma(\ell)$, i.e., $\sigma(\xi)/\sigma_\infty$. (The $\ell(T)$ dependence should be determined independently, as was done for example in [8].) The ratio $\sigma(\xi)/\sigma_\infty$, according to (6), determines $\phi^*(\xi)$. By inverting the function $\phi^*(\xi)$ obtained in this manner, i.e., by simply plotting ξ against ϕ^*, we obtain, in accordance with (7), the function $\Phi = \Phi(\phi^*)$, and consequently the function of interest to us:

$$1 - q(\phi) = \Phi(\phi)/\phi.$$

In conclusion, we note the following two circumstances. Greatest interest attaches to the region where the reflection differs noticeably from the diffuse

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[1)] A similar calculation yields for a wire $\sigma/\sigma_\infty \sim \phi^{*2} + \xi$, $\xi = d/\ell$, where d is the wire diameter, and ϕ^* is as before the characteristic reflection angle determined from (7).

one, i.e., $q(\phi) \sim 1$, and thus $\phi \leq h/\delta p_F \ll 1$ (δ is the characteristic dimension of the microscopic surface roughnesses, and h/p_F is the de Broglie wavelength of the electron). According to (7), this means $\xi \lesssim (h/\delta p_F) \ll 1$. At such extremely small ξ, the here-employed concepts of path time and mean free path, and for the same reason also $q(\phi)$, have an exact meaning [9] (see also [8]). Further, since the region $\phi \ll 1$ is of importance in (3), the integration in (3) can be formally extended to infinity. Relation (3) then takes the form of the Wiener-Hopf equation for the function $\phi = \phi(\Phi)$, so that this function can be determined from the known function $\phi(\xi)$. Such a more exact analysis, however, is hardly meaningful under our assumption that the dispersion is isotropic.

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AMPLIFICATION OF ELECTROMAGNETIC FIELD IN TOTAL INTERNAL REFLECTION FROM A REGION OF INVERTED POPULATION

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The solution of the problem of passage and reflection of electromagnetic waves at an interface of two media, each having its own dielectric constant $\varepsilon_m = \varepsilon_m' + i\varepsilon_m''$ (m = 1, 2), is determined by Maxwell's equation together with the conditions of continuity on the interface and the radiation condition at infinity.

It is convenient to represent the general solution of the problem in the form of a superposition of plane waves, the reflection and transmission coefficients of which are given by particularly simple expressions in the case of a plane boundary. But the imaginary part of the dielectric constant enters quadratically in these expressions, and it may therefore turn out that the coefficient of reflection from the interface is independent of whether the reflecting medium is absorbing ($\varepsilon' > 0$) or inverted ($\varepsilon' < 0$). This is indeed the case if the wave incidence angle is smaller than the angle of total internal reflection. In the case of "total internal reflection" from an inverted-population region, however, the reflection coefficient turns out to be larger than unith! This is the consequence of selecting the type of the solution of Maxwell's equation as a function of the sign of ε''.

Let us assume for simplicity that the plane wave is incident from a transparent medium ($\varepsilon_1' = \varepsilon_0$, $\varepsilon_1'' = 0$) on a plane boundary of an inverted medium