

where  $\theta_0$  is the specific (per unit length) Faraday rotation angle [3].

Thus, we see from (6), (7), and (4) that, unlike in [1], the dependence of the refractive index on the propagation direction of an electromagnetic wave relative to the current  $J_0$  is determined by the nonparabolicity of the energy band. If there is no magnetic field, then  $\Delta n_{II} = 0$  and  $\Delta n_I = \Delta n_{III} = (\omega/c)\delta F = \Delta n$ . In the extremely degenerate case,  $\Delta n$  can be represented in the form

$$\Delta n = i_0 \frac{8\pi e p^2(\epsilon)}{5\omega^2 c m^3(\epsilon)} \frac{d m(\epsilon)}{d \epsilon} \bigg|_{\epsilon = \epsilon_F} \quad (8)$$

An estimate based on (8) gives for the experimental conditions of [1] (n-InAs,  $T = 80^\circ\text{K}$ , carrier density  $5.5 \times 10^{17} \text{ cm}^{-3}$  and  $\lambda = 3.39 \mu$ ) a value of  $\Delta n$  which is smaller by a factor 2 - 3 than the observed value. This is apparently due to the fact that interband effects play an important role in this experiment (the energy of the light quantum is close to the width of the forbidden band). On the other hand, in the region of longer wavelengths, where interband effects are insignificant, measurements of  $\Delta n$  make it possible, in principle, to determine a number of important plasma parameters in the conduction band (see formula (8)).

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#### STATIONARY TURBULENCE OF A PARAMETRICALLY UNSTABLE PLASMA

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The theoretically predicted [1] phenomenon wherein the energy of a powerful radiation flux is transferred to a plasma at anomalously high speed is due to the onset of parametric instability of the plasma. Of great importance to the full understanding of this phenomenon is the development of concepts concerning the turbulent state produced in the plasma during the evolution of the parametric instability. The results obtained to date in the theory of stationary turbulence are connected with allowance for the nonlinear interaction of the growing plasma perturbations, this interaction being due to stimulated scattering of the waves by the particles. It is precisely the scattering of ion-acoustic waves by ions which made it possible to obtain a stationary level of fluctuations of a turbulent non-isothermal plasma in the field of a strong pump wave [2] (see also [3]). A stationary turbulence level was subsequently obtained under somewhat different conditions [4, 5] via scattering of waves by particles.

The stimulated scattering of waves by particles, which were taken into account in [2 - 5], are by far not the only nonlinear processes of interaction between waves and plasma particles in the same approximation of the expansion in powers of the plasma-fluctuation intensity. We wish to call attention here to the role that the nonlinear frequency shift of the plasma oscillations plays in the establishment of the stationary turbulence level of a parametrically unstable plasma. The stabilizing effect of such a frequency shift is

demonstrated using as an example aperiodic parametric instability, which, unlike the instability of the type of Raman scattering, or, which is the same, the decay of the pump wave into a high-frequency plasma wave and a low-frequency ion-acoustic wave, has apparently a more universal significance.

Aperiodic perturbations in a parametrically excited plasma grow together with the plasma oscillation at the frequency  $\omega_0$  of the pump wave. To explain the role of the nonlinear processes in the stabilization of parametric instability, we therefore consider first the nonlinear dispersion equation of high-frequency plasma oscillations

$$\epsilon(\omega, \vec{k}) + \int \frac{d\vec{k}'}{(2\pi)^3} W(\vec{k}') Q(\vec{k}, \vec{k}') = 0. \quad (1)$$

Here  $\epsilon(\omega, \vec{k})$  is the linear longitudinal dielectric constant of the plasma,  $W(\vec{k})$  is the spectral energy density of the plasma oscillations, and the kernel  $Q(\vec{k}, \vec{k}')$  is given by

$$Q(\vec{k}, \vec{k}') = 2\pi \frac{e^2}{m^2} \left( \frac{\vec{k} \vec{k}'}{k k'} \right)^2 \frac{(k - k')^2}{(\omega - \omega')^2} \frac{\delta\epsilon_e(\omega - \omega', k - k')}{\epsilon(\omega - \omega', k - k')} \times \\ \times [1 + \delta\epsilon_i(\omega - \omega', k - k')],$$

where  $e$  and  $m$  are the charge and mass of the electron;  $\omega, \omega'$  and  $\vec{k}, \vec{k}'$  are the frequencies and wave vectors of the interacting plasma oscillations,  $\delta\epsilon_e$  and  $\delta\epsilon_i$  are the partial dielectric constants of the electronic and ionic components of the plasma, so that  $\epsilon = 1 + \delta\epsilon_e + \delta\epsilon_i$ . In view of the smallness of the nonlinear frequency shift  $\delta\omega(\vec{k})$ , we obtain from the real part of Eq. (1) ( $\omega_p$  is the plasma frequency and  $r_{De}$  is the Debye radius of the electrons):

$$\omega = \omega(\vec{k}) + \delta\omega(\vec{k}); \quad \omega(\vec{k}) = \omega_p \left( 1 + \frac{3}{2} k^2 r_{De}^2 \right); \\ \delta\omega(\vec{k}) = -\frac{1}{2} \omega(\vec{k}) \int \frac{d\vec{k}'}{(2\pi)^3} W(\vec{k}') \operatorname{Re} Q(\vec{k}, \vec{k}'). \quad (2)$$

The imaginary part of (1) gives the usual expression for the linear damping decrement  $\tilde{\gamma}(\vec{k})$  of the high-frequency plasma waves ( $\nu_{ei}$  is the frequency of the electron-ion Coulomb collisions, and  $v_{Te}$  is the thermal velocity of the electrons)

$$\tilde{\gamma}(\vec{k}) = \frac{\nu_{ei}}{2} + \sqrt{\pi/8} \frac{\omega_p}{(k r_{De})^3} \exp[-(1/2)(\omega_0/kv_{Te})^2]$$

and the nonlinear increment  $\delta_Y(\vec{k})$  to the damping, which in general may be comparable with  $\tilde{\gamma}$ :

$$\delta_Y(\vec{k}) = \frac{1}{2} \omega(\vec{k}) \int \frac{d\vec{k}'}{(2\pi)^3} W(\vec{k}') \operatorname{Im} Q(\vec{k}, \vec{k}'). \quad (3)$$

Expressions (2) and (3) can be added to the formulas of the usual theory of the thresholds of aperiodic parametric plasma instability [6], to the frequency of the plasma oscillations and to their damping decrement, respectively. Following this addition, the condition that the increment of the parametric instability vanish leads to a relation that determines the stationary turbulence level:

$$(\tilde{\gamma} + \delta\gamma)^2 + (\Delta\omega_0 - \delta\omega)^2 = -\frac{\omega_p}{4} \frac{(\kappa E)^2}{k^2(r_{De}^2 + r_{Di}^2)} (\Delta\omega_0 - \delta\omega). \quad (4)$$

Here  $\vec{r}_E \equiv (\vec{e} \cdot \vec{E} / m\omega_0^2)$  is the amplitude of the electron oscillations in the pump-wave field with electric field intensity  $\vec{E}_0$ ,  $r_{Di}$  is the Debye radius of the ions, and  $\Delta\omega_0 = \omega_0 - \omega(\vec{k}) < 0$ . From (4) it follows that if the electric field of the pump wave exceeds a threshold value  $E_{thr}$  determined by the linear theory of parametric resonance [6] ( $N_e$ ,  $N_i$ , and  $T_e$ ,  $T_i$  are the electron and ion densities and temperatures, and  $\kappa$  is Boltzmann's constant)

$$E_{thr}^2 = 16\pi \frac{\nu_{ei}}{\omega_p} (N_e \kappa T_e + N_i \kappa T_i),$$

then nonlinear stabilization of the aperiodic parametric instability is possible also when  $\tilde{\gamma} \gg \delta\gamma$ , i.e., only as a result of the nonlinear frequency shift  $\delta\omega$  of the high-frequency plasma oscillations. We confine ourselves to this simple case. Such a nonlinear stabilization is made possible by the negative value ( $\delta\omega < 0$ ) of the nonlinear frequency shift (2), which increases, in accord with (4), the negative frequency deviation  $\Delta\omega_0$ .

Not far from the threshold, the region of wave vectors corresponding to aperiodic parametric instability, is limited by a narrow interval of values of the wave numbers near  $k_0$  (we assume that  $\tilde{\gamma}(\vec{k}) \approx \nu_{ei}/2$ )

$$k_0 = \frac{1}{r_{De}} \sqrt{\frac{\nu_{ei} + 2(\omega_0 - \omega_p)}{\omega_p}} \quad (5)$$

and by the narrow cone of directions with axis along the pump electric-field intensity  $\vec{E}_0$ . Therefore under the easily attained conditions ( $v_{Ti}$  is the thermal velocity of the ions)

$$(k - k')^2 r_{Di}^2 < 1, \quad \frac{3}{2} \omega_p (k^2 - k'^2) r_{De}^2 < |k - k'| v_{Ti}$$

the nonlinear frequency shift of the plasma oscillations in the near-threshold region

$$\delta\omega(k) = -\frac{\omega_p}{8} \left( \frac{eE}{m\omega_p^2} \right)^2 \frac{1}{r_{De}^2 + r_{Di}^2}$$

does not depend on the wave vector and is determined by the effective intensity of their electric field  $E$ :

$$\int \frac{d\mathbf{k}}{(2\pi)^3} W(\mathbf{k}) = \frac{E^2}{8\pi}.$$

The value of the field  $E$ , characterizing the level of the stationary turbulence, is obtained by solving Eq. (4), in which it is necessary to take into account the fact that for the wave number (5) the frequency deviation  $\Delta\omega_0 = -(\nu_{ei}/2)$  is determined by the damping decrement  $\tilde{\gamma}$ . It turns out here that near the threshold the effective intensity of the electric field of the high-frequency plasma oscillations is given by

$$E^2 = (E_0^4 - E_{thr}^4)^{1/2} - (E_0^2 - E_{thr}^2). \quad (6)$$

Taking into account the connection between the levels of the aperiodic perturbation and of the high-frequency plasma oscillations in parametric wave buildup, we can easily obtain with the aid of (6) the value of the effective electric field intensity  $E_a$  of the aperiodic perturbation in the plasma ( $e_i$  is the ion charge):

$$E_a^2 = -\frac{4}{3} E_{thr}^{3/2} (E_0 - E_{thr})^{1/2} \frac{e_i}{|e|} T_e T_i^2 \frac{\nu_{ei}}{\omega_0} \left[ \frac{\nu_{ei}}{\omega_0} + \frac{2(\omega_0 - \omega_p)}{\omega_0} \right] \times \\ \times \left( T_i + \frac{e_i}{|e|} T_e \right)^{-3}$$

For a larger excess of  $E_0$  above the threshold  $E_{thr}$ , the effective electric field intensity  $E$  of the high-frequency plasma oscillations in the discussed turbulent state of the plasma, represented by the narrow region of the space of the near-threshold wave vectors, does not exceed  $E_{thr}$ . Equation (4) can be used to describe the stationary turbulent state also in a much larger region of wave-vector space (for example, at a considerable distance from threshold).

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