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CRITICAL CHARGE IN THE TWO-CENTER PROBLEM

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Submitted 21 July 1972

ZhETF Pis. Red. 16, No. 6, 355 - 358 (20 September 1972)

The energy of an electron in the field of a point Coulomb center Ze is equal to (for the $1s$ level):

$$\epsilon_0 = \sqrt{1 - (Z\alpha)^2}, \quad (1)$$

where $\hbar = c = m_e = 1$ and $\alpha = 1/137$. This expression has a singularity at $Z = 137$. As noted by Pomeranchuk and Smorodinskii, allowance for the finite dimensions of the nucleus eliminates the Coulomb singularity, and formula (1) continues into $Z > 137$. The value $Z = Z_c$ at which the $1s$ level joins the lower continuum is called the critical charge of the nucleus. Quantum electrodynamics leads to a number of characteristic predictions in the region $Z > Z_c$. The main effect is the emission of positron by a "bare" nucleus, i.e., a nucleus with unfilled K-shell (for details see [2 - 4]). For a spherical nucleus with radius $R \sim 10 F$ we have numerically $Z_c \approx 170$ (see [2, 5]), which is far from the region of presently known heavy elements. For this reason, it is necessary to resort to another method of obtaining supercritical fields, namely, in a collision between two heavy nuclei with charges Z_1 and Z_2 , such that $Z_1 + Z_2 > Z_c$ (such a possibility was first discussed in [6]; see also [4, 7]). When considering this effect, it is necessary to find first the "critical" distance $R_c = R_c(Z_1, Z_2)$ in the relativistic problem of two centers, i.e., that distance R between charges, for which the ground-state level of the quasimolecule (Z_1, Z_2, e) drops to the limit $\epsilon = -1$. We present here the results of such calculations. We confine ourselves to the simplest case $Z_1 = Z_2 = Z$.

At $Z < 137$, the radius of the nucleus is immaterial, for there is no "falling to the center" in a Coulomb field with $Z < 137$. In other words, the break-up of the total charge $2Z$ into two parts separated by a finite distance R is in itself sufficient for a regularization of the problem. The curve of the level $\epsilon_0 = \epsilon_0(Z)$ in the two-center problem reaches $\epsilon = -1$ without having on its path singularities of the type (1). For this reason, we shall regard the nuclei as point-like (the radii of heavy nuclei are $r_0 \sim 8 F$, which is much less than the K-orbit radius $r_K = (1 + 2\sqrt{1 - \zeta^2})/2\zeta = 700 F$ for uranium).

The actual excess of $2Z$ over Z_c is small (for uranium nuclei, $\delta = (2Z - Z_c)/Z_c = 0.08$, and for Cf + Cf we have $\delta = 0.15$), and therefore $R < \hbar/m_e c = 1$. In the region $\delta \ll 1$ we can obtain for R_c a simple formula by using the method of matching the asymptotic expansions. We shall explain the main idea with spinless particles as an example.

The form of the wave function ψ_0 at small and large distances is determined from the Klein-Gordon equation [7]. Near the nuclei we have

$$\psi_0(\xi, \eta) = (\xi^2 - \eta^2)^{-\sigma/2}, \quad \sigma = 1 - \sqrt{1 - \zeta^2}, \quad (2)$$

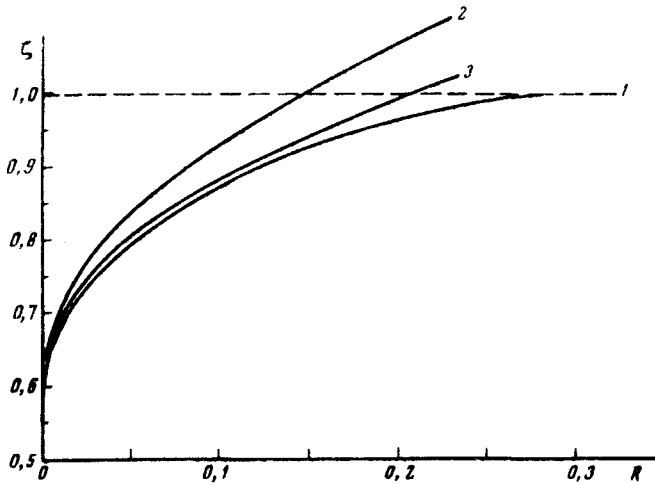


Fig. 1. Critical charge for scalar particles (R is in units of \hbar/mc).

whereas in the region $r_1, r_2 \gg R$ the nuclei can be regarded as one unit, and therefore

$$\psi_0 = r^{-1/2} K_{i\nu}(\sqrt{8\zeta r}), \quad \nu = \sqrt{4\zeta^2 - 1}. \quad (3)$$

Here $\zeta = 2Z/137$, $K_{i\nu}(x)$ is a Macdonald function, and $\xi = (r_1 + r_2)/R$ and $\eta = (r_1 - r_2)/R$ are elliptic coordinates. Under the condition $R \ll 1$, there is an intermediate region $R \ll r \ll 1$, in which $\xi \gg 1$, η and the asymptotic forms (2) and (3) are joined together. It can be shown that in this region of r the wave function depends mainly on the variable ξ , and furthermore in power-law fashion. Equating the exponents for the external and internal solution with energy $\epsilon = -1$, we obtain an equation for the critical distance between the nuclei:

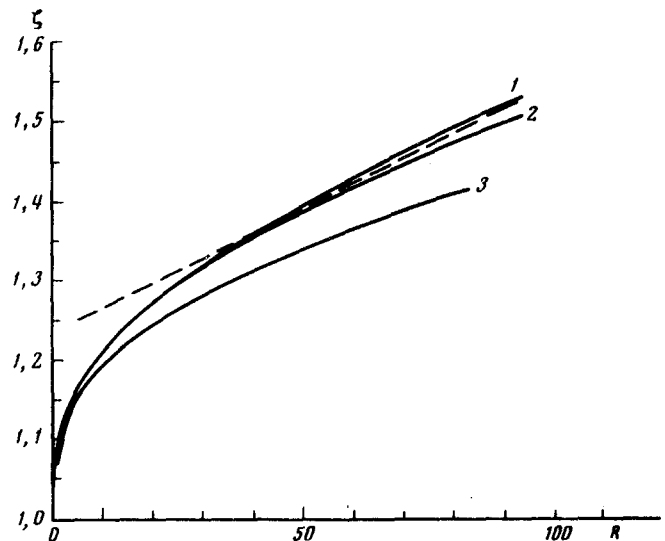
$$R_c = \frac{1}{\zeta} \exp \left\{ -\frac{2}{\nu} \left[\text{arc ctg} \frac{1 - 2\sqrt{1 - \zeta^2}}{\sqrt{4\zeta^2 - 1}} - \arg \Gamma(1 + i\nu) \right] \right\} \quad (4)$$

(see curve 1 of Fig. 1). It is obvious that the same curve gives the critical charge ζ_c at a fixed distance R between nuclei. For comparison, Fig. 1 shows the curves obtained from the exact solution [2] for ζ_c in the case of a spherical nucleus with a radius equal to $R/2$ (curve 2 corresponds to a surface distribution of the charge, and curve 3 to a volume distribution). It is easy to prove rigorously with the aid of the variational principle [7] that curve 2 should lie higher than the true curve $\zeta_c(R)$ in the two-center problem. The locations of curves 1 and 2 satisfy this requirement. Attention is called to the proximity of curves 1 and 3, meaning the approximate equivalence (in the sense of Z_c) of the following two distributions: 1) two point-like centers Z separated by a distance R ; 2) a spherical nucleus with diameter R and charge $2Z$ uniformly distributed over its volume.

In the case of a spin $s = 1/2$, the calculations are analogous albeit more cumbersome. We present the final result:

$$R_c = \frac{1}{\zeta} \exp \left\{ -\frac{1}{g} \left[\text{arc ctg} \frac{\zeta^2 - 1 - \sqrt{4 - \zeta^2}}{\sqrt{\zeta^2 - 1}(1 + \sqrt{4 - \zeta^2})} - \arg \Gamma(1 + 2ig) \right] \right\}, \quad (5)$$

Fig. 2. Critical charge for electrons (R is in Fermi units).



where now $g = \sqrt{\zeta^2 - 1}$, $1 < \zeta < 2$. This relation is plotted in Fig. 2 (the curves 1 - 3 have the same meaning as in Fig. 1). Of course, for $R < 20$ F, when the charges coalesce, formula (5) no longer holds. We should obtain here the same value $\zeta_c = 1.25$ as for one extended nucleus. In the region of values of R of interest to us, the dependence of Z_c on R is approximately linear (see the dashed curve in Fig. 2). From this we get $R_c = 40$ F for uranium nuclei, and $R_c = 60$ F for Cf + Cf.

In conclusion, a few words concerning the accuracy of the asymptotic formulas (4) and (5). To verify them we solved by the same method two problems for which the exact solution is known: 1) bound level in a field of two δ -potentials; 2) critical charge Z_c for one extended nucleus. In both cases, perfectly satisfactory agreement with the exact solution was obtained. Thus, for the critical charge $Z_c = 137\zeta_c$ in a spherical nucleus with radius r_0 we obtain the formula

$$r_0 = \frac{1}{2\zeta} \exp \left\{ -\frac{1}{g} \left[\text{arc ctg} \left(-\frac{\xi}{g} \right) - \arg \Gamma(1 + 2ig) \right] \right\} \quad (6)$$

($\xi = [rG'/G]_{r=r_0}$ is the logarithmic derivative of the wave function at the edge of the nucleus), which results in an error not larger than 1% in the region $r_0 \sim 10$ F.

The author is sincerely grateful to S.S. Gershtein for useful discussions during the course of the work, and also to L.B. Okun' and A.M. Perelomov for a discussion of the results.

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JUMP OF SPECIFIC HEAT OF LIQUID He³ AT 2.65×10^{-3} °K

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Submitted 10 August 1972

ZhETF Pis. Red. 16, No. 6, 358 - 360 (20 September 1972)

Osheroff, Richardson, and Lee [1] traced the cooling of He³ by adiabatic crystallization to infralow temperatures near 0.001°K. A kink with a change of slope by a factor 1.8 was observed on the plot of the pressure against time at a temperature ~ 0.0027 °K (point A). This was explained by the authors as the production of a new solid phase via a first-order phase transition. The explanation given in [1], however, is debatable.

First, the behavior of a platinum NMR thermometer is characteristic of the superheat due to the Kapitza jump, in view of the constant heat release, and does not require the formation of a new solid phase for its explanation.

Second, the transition at A is very abrupt (pressure interval 3×10^{-4} atm); under the condition of fast passage of the pressure interval to the maximum (less than 15 minutes), it is reproducible under compression and expansion, and does not depend on the rate of change of the volume and on the amount of solid He³ in the chamber. Furthermore, in view of the large relaxation times in solid He³, as found from crystallization studies, the main cooled reservoir is the liquid He³. This indicates that the solid helium in the chamber is far from temperature equilibrium, and at the same time all the features of the transition are completely independent of the degree of non-equilibrium of the solid.

It can be more readily assumed that the phenomenon at point A occurs in the liquid He³, which has a short thermal-relaxation time. At 0.003°K we have

$$\tau = \frac{\ell^2 C_p}{\kappa V} = 0.3 \text{ sec,}$$

where $\ell = 2$ mm is the mean distance to nearest liquid-solid He³ interface, $C_p = 10^{-1}$ J/mole-°K is the molar specific heat [3], $\kappa = 5 \times 10^{-4}$ W/cm-°K is the thermal conductivity [4], and $V = 25$ cm³/mole is the molar volume. At the indicated cooling rate and at the temperature interval 10^{-5} °K given in [1], in which the transition takes place, the transition time is ~ 2 sec, which coincides with the relaxation time in the liquid.

The assumptions that (i) the transition occurs in the liquid, and (ii) the solid is strongly thermally insulated in the fast process, in view of the low thermal conductivity, lead to the following conclusion: at a constant rate of volume change, i.e., at a constant rate of heat absorption, the kink on the time dependence of the pressure $P(t)$, meaning on the time dependence of the temperature, corresponds to a jump in the specific heat of the liquid.

It must be recognized that owing to the pressure dependence of the molar volumes of the liquid and solid He³, a change in the temperature causes an additional change in the volume of the solid phase. A temperature rise corresponds to absorption of heat, and the liquid-solid He³ interface represents an additional specific heat