POSSIBLE UNIVERSAL NEUTRINO INTERACTION

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Much attention is being paid of late in elementary-particle physics to the possible degeneracy of vacuum, and the ensuing spontaneous breaking of one symmetry or another. The most direct consequence of vacuum degeneracy is the occurrence of the zero-mass particles called Goldstone particles [1].

Of all the presently known elementary particles, only the neutrino, photon, and graviton have zero mass. The last two, however, correspond to gauge fields and apparently do not require vacuum degeneracy for their description. The neutrino is therefore the only particle whose existence may be directly connected with vacuum degeneracy.

We wish to point out here that the hypothesis that the neutrino is a Goldstone particle leads to a definite type of neutrino interaction both with other neutrinos and with all other particles. The interaction is fully defined by a single phenomenological coupling constant and is in this sense universal.

To determine the type of symmetry whose spontaneous violation causes the degeneracy of vacuum and the corresponding properties of the neutrino as a Goldstone particle, let us consider the symmetry properties of the equation for the free neutrino

$$i\sigma_\mu \frac{\partial}{\partial x_\mu}\psi = 0. \tag{1}$$

This equation is invariant both with respect to the Poincare group in chiral transformations, and with respect to shifts in spinor space, i.e., with respect to transformations of the type

$$\begin{aligned} \psi &\to \psi' = \psi + \zeta, \\ x &\to x' = x, \end{aligned} \tag{2}$$

where ζ is a constant spinor that anticommutes with ψ.

We retain the character of the transformations of x_μ and ψ in the transformations of the Poincare group, and replace the transformations (2) by the transformations

$$\begin{aligned} \psi &\to \psi' = \psi + \zeta, \\ \psi^+ &\to \psi^{+\prime} = \psi^+ + \zeta^+, \\ x_\mu &\to x_\mu' = x_\mu - \frac{\alpha}{2i}(\zeta^+\sigma_\mu\psi - \psi^+\sigma_\mu\zeta). \end{aligned} \tag{3}$$

The resultant structure with ten commuting and four anticommuting parameters has the structure of a group[1)] and is the only possible generalization of (2)

[1)] Lie groups with commuting and anticommuting parameters were recently considered by Berezin and Kats [2].

without introducing additional group parameters.

The constant a in the transformations (3) is arbitrary and has the dimension of length raised to the fourth power. We postulate that the equations for the neutrino with allowance for the interaction are invariant against the transformations (3). We assume also that the terms of the interaction contain the minimum number, compatible with the invariance requirement, of field derivatives.

To construct the phenomenological action integral under the foregoing assumptions, it suffices to use the following differential forms, which are invariant against the transformations (3)

$$\omega_\mu = dx_\mu + \frac{a}{2i}(\psi^+\sigma_\mu d\psi - d\psi^+\sigma_\mu\psi). \tag{4}$$

The action integral invariant against transformations (3) and the transformations of the Poincare group is given by

$$S = \frac{1}{a}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 , \tag{5}$$

where Λ denotes an external product. The expression (5) corresponds to a certain four-dimensional volume in the group-parameter space.

If the 4-volumes are defined by specifying the function $\psi = \psi(x)$, the action integral (5) can be written in the following more usual form:

$$S = \frac{1}{a}\int |W|\, d^4x , \tag{6}$$

where $|W|$ is the determinant of the matrix W

$$W_{\mu\nu} = \delta_{\mu\nu} + a T_{\mu\nu} ,$$
$$T_{\mu\nu} = \frac{1}{2i}(\psi^+\sigma_\mu\partial_\nu\psi - \partial_\nu\psi^+\sigma_\mu\psi). \tag{7}$$

It follows from (6) and (7) that the action integral as a function of the tensor T takes the form

$$S = \int\left[\frac{1}{a} + T_{\mu\mu} + \frac{a}{2}(T_{\mu\mu}T_{\nu\nu} - T_{\mu\nu}T_{\nu\mu}) + \frac{a^2}{3!}\sum_p(-1)^p T_{\mu\mu}T_{\nu\nu}T_{\rho\rho} + \frac{a^3}{4!}\sum_p(-1)^p T_{\mu\mu}T_{\nu\nu}T_{\rho\rho}T_{\sigma\sigma}\right]d^4x, \tag{8}$$

where the summation over p corresponds to a sum over all the permutations of the second indices in the products of the tensors T.

The term with $T_{\mu\mu}$ corresponds to the kinetic terms, and the terms with products of two, three, and four tensors T describe interactions in which four, six, and eight fields, respectively, participate. The degrees of the field derivatives in the interaction terms are determined by the number of factors T.

The neutrino interaction with other fields can be determined in a manner that is invariant with respect to the transformations (3).

Thus, for example, the action integral for a Dirac particle is given by

$$S = \int \Big[R_{\mu\mu} + a\,(R_{\mu\mu} T_{\nu\nu} - R_{\mu\nu} T_{\nu\mu}) + \frac{a^2}{2} \sum_{p} (-1)^p R_{\mu\mu} T_{\nu\nu} T_{\rho\rho}$$

$$+ \frac{a^3}{3!} \sum_{p} (-1)^p R_{\mu\mu} T_{\nu\nu} T_{\rho\rho} T_{\sigma\sigma} + m \bar{\phi} \phi \, |W| \Big] d^4 x \,, \qquad (9)$$

where

$$R_{\mu\nu} = \frac{1}{2i} (\bar{\phi} \gamma_\mu \partial_\nu \phi - \partial_\nu \bar{\phi} \gamma_\mu \phi) \qquad (10)$$

and the tensor T and the determinant $|W|$ are defined in (7).

Weak interactions can be included in the scheme under consideration by introducing gauge fields for the approximate unitary symmetry group of the neutrino and other leptons. The electromagnetic interaction for charged leptons is introduced simultaneously. The weak and electromagnetic interactions are turned on simultaneously by the well-known mechanism of spontaneous breaking of the unitary-group symmetry [3]. In the zero-lepton-mass limit, the unitary symmetry is exact. To obtain the action integral for the leptons in the unitary-symmetry limit, it suffices to regard in (3), (4), and (7) the spinor products as invariant products of unitary multiplets. It is also possible to add to formulas (3), (4), and (7) the terms for leptons with opposite chirality. The unitary groups for states with different chirality need not necessarily coincide in this case.

The gauge fields can be introduced into the so-generalized action integral in a manner covariant with respect to the transformations (3).

The gravitational interaction can be introduced into the scheme in analogous fashion, by introducing gauge fields corresponding to the Poincare group.

We note that if we introduce also gauge fields corresponding to the transformations (3), then, as a consequence of the Higgs effect [4], a massive gauge field with spin 3/2 arises, and the Goldstone particles with spin 1/2 vanish.

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