The nuclei become self-polarized because the electrons are artificially kept in a non-oriented state. Indeed, we have assumed that the fluctuation of the nuclear polarization and the appearance of an effective magnetic field are not accompanied by a corresponding change of the electron orientation. Such conditions can be created in practice, for example, in semiconductors when non-equilibrium non-oriented carriers are produced by light, or by injection of carriers through a junction, if their lifetime τ is much shorter than the spin-relaxation time τ_s. Otherwise the critical temperature becomes lower than the value given in (5), owing to the additional factor $\tau_s/(\tau + \tau_s)$. Another effect that lowers T_c may be the spin-lattice relaxation of the nuclei. The corresponding decrease of T_c is described by the factor $\gamma/(\nu + \gamma)$, (leakage factor) where ν is the reciprocal time of longitudinal nuclear relaxation.

In the absence of an external magnetic field, the direction of the macroscopic nuclear magnetization at $T < T_c$ is random (in the simple model under consideration). It can be shown that a weak external field $H \ll kT_c/(g\mu_B)$, without changing the general picture of the phenomenon, makes the orientation of the macroscopic magnetization of the nuclei along the magnetic field the most probable.

[1] L.D. Landau and E.M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963 [Addison-Wesley, 1965].
[2] M. Gueron, Phys. Rev. 135, A200 (1964).
[3] A. Willig and B. Sapoval, Eleventh International Conference on Physics of Semiconductors (Abstracts), P. 31, Warszawa, 1972.

SINGULARITY OF THE BEHAVIOR OF A SUPERCONDUCTING FILM IN AN ALTERNATING FIELD OF FREQUENCY CLOSE TO THRESHOLD

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We consider the behavior of a superconducting film in an external microwave field whose amplitude in a real experimental situation is small compared with the critical value. Let also the frequency of this field be close to the threshold $2\Delta_0$, where Δ_0 is the equilibrium value of the gap. The linear properties of the superconductors at such frequencies have been thoroughly investigated [1], and we are therefore interested in nonlinear effects.

To this end, we obtain the response of the ordering parameter $\Delta(t)$ which is a small increment to the equilibrium value Δ_0 to an external alternating field. It is expressed in terms of the Green's function

$$\Delta_\omega = \lambda \int \frac{d\epsilon}{4\pi i} \int \frac{d^3p}{(2\pi)^3} F_{\epsilon\epsilon-\omega}. \qquad (1)$$

Expanding the function $F_{\varepsilon\varepsilon-\omega}$ in terms of Δ_ω and the alternating-field potentials $\vec{A}_\omega$, we rewrite (1) in an integral form

$$\int K(\omega\omega_1)\,\Delta_{\omega_1} d\omega_1 = \int a(\omega\omega_1)\,\mathbf{A}_{\omega_1}\mathbf{A}_{\omega-\omega_1} d\omega_1 + \dots \qquad (2)$$

the right-hand side of which is a series in powers of the field intensity, and K and $\underline{a}$ are certain kernels. Without allowance for the dependence of K on the

potential, we would obtain for Δ_ω a series in powers of $\vec{A}^2$, which corresponds to perturbation-theory allowance for the nonlinearity. Such a case was considered in [2], where $K(\omega\omega_1) = (\omega^2 - 4\Delta^2)\delta(\omega_1)$.

We now take into account the dependence of the kernel K on the alternating field. Equation (2) now takes the form

$$(\omega^2 - 4\Delta^2)\Delta_\omega + \int \phi A_{\omega_1} A_{\omega_2} \Delta_{\omega-\omega_1-\omega_2} d\omega_1 d\omega_2 = \int \sigma A_{\omega_1} A_{\omega-\omega_1} d\omega_1. \quad (3)$$

By virtue of the foregoing, we are interested only in the solution of the homogeneous equation, and discard therefore the right-hand side of (3). In the case of a monochromatic external field, the left-hand side will contain a term of the type $\vec{A}_\omega \vec{A}_\omega \Delta_{-\omega}$, which corresponds, as it were, to modulation of the natural frequency $\omega_0 = 2\Delta$ at a frequency 2ω. If 2ω is close to double the natural frequency $2\omega_0$, then parametric resonance becomes possible under certain conditions [3].

Let us explain this possibility. To this end it is necessary first to calculate the kernel ϕ, which is proportional to the Green's function $F_{\varepsilon\varepsilon-\omega}$ containing the two potentials $\vec{A}$ and one increment to the gap. At an arbitrary temperature, we use for the calculations the procedure of [4], which yields Green's functions integrated with respect to $\xi = v(p - p_0)$. Integrating over the impurities, whose concentration is assumed large, so that l_{tr} is less than either the coherence length or the film thickness, we obtain for $|\omega - 2\Delta| \ll \Delta$

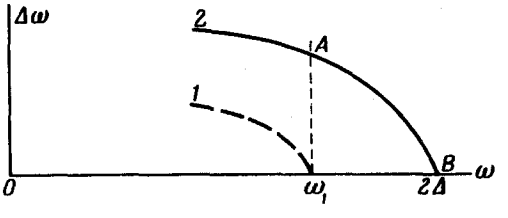
$$f_{\varepsilon\varepsilon-\omega} = 4\pi i \left[D\left(\frac{e}{c}\right)^2 A^2 \Delta \right]_\omega \frac{4\Delta^2}{(2\varepsilon-\omega)^2} \operatorname{th}\frac{|\varepsilon|}{2T} \operatorname{th}\frac{|\varepsilon-\omega|}{2T} \times$$
$$\times \left[\frac{1}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{\Delta^2-(\varepsilon-\omega)^2}} + \frac{1}{\sqrt{\Delta^2-\varepsilon^2}\sqrt{(\varepsilon-\omega)^2-\Delta^2}} \right],$$

where $D = v l_{tr}/3$ is the diffusion coefficient, and the entire expression differs from zero only in the region of the real roots.

Equation (3) thus takes the form (at $T \ll \Delta$)

$$(\omega^2 + 2i\gamma\omega - 4\Delta^2)\Delta_\omega + \Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2} \frac{A_\omega A_{-\omega}}{A_1^2} \Delta_\omega + \Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2} \times$$
$$\times \frac{A_\omega A_\omega}{A_1^2} \Delta_{-\omega} = 0, \quad (4)$$

where $(32/\pi)D(e/c)^2 A_1^2 = \Delta$ (A_1 is the order of magnitude of the critical field of a bulky sample), and where allowance is made for the damping $\gamma \sim \lambda\Delta(\Delta/\omega_D)^2 (T/\Delta)^{7/2}$ calculated in [2]. It is seen from (4) that the role of the field reduces not only to a modulation of the natural frequency of the oscillations, but also to its renormalization. In analogy with [3], it can be shown that the zero-order solution of (4) is unstable at a specified field intensity in a certain frequency interval to be indicated below. The establishment of the final oscillation amplitude is therefore



connected with nonlinearity effects that must be taken into account in (4).

The higher orders of Δ_ω are calculated in analogy with the determination of the term with $\vec{A}^2$. Without dwelling on the intermediate steps, we present the final equation

$$(\omega^2 + 2i\gamma\omega - 4\Delta^2)\,\Delta_\omega + \Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2}\frac{A_\omega A_{-\omega}}{A_1^2}\Delta_\omega + \Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2}\times$$

$$\times\,\frac{A_\omega A_\omega}{A_1^2}\,\Delta_{-\omega} + (4\Delta^2-\omega^2)\,f\left(\frac{\Delta_\omega\Delta_{-\omega}}{4\Delta^2-\omega^2}\right) = 0, \tag{5}$$

where $f(p) = p$ at $p \ll 1$.

To find the solution we put $A(t) = A\cos\omega t$, and seek $\Delta(t)$ in the form $\cos(\omega t + \alpha)$. Then, getting rid of the imaginary part of (5), we obtain

$$\omega^2 - 4\Delta^2 + \Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2}\frac{A^2}{A_1^2} \pm \sqrt{\left[\Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2}\frac{A^2}{A_1^2}\right]^2 - (4\gamma\Delta)^2} +$$

$$+\,(4\Delta^2-\omega^2)\,f\left(\frac{\Delta_\omega\Delta_{-\omega}}{4\Delta^2-\omega^2}\right) = 0. \tag{6}$$

It is clear therefore that buildup of oscillations is possible only when the field intensities exceeds a certain limit connected with the damping of the natural oscillations. We shall assume this condition to be satisfied and thus neglect the quantity γ in (6).

Approximating the function f with good accuracy by a linear one, we obtain the two branches of the solution of (6)

$$\Delta_\omega^{(1)} = \left[4\Delta^2 - \omega^2 - 2\Delta^2\left(\frac{\Delta}{2\Delta-\omega}\right)^{3/2}\frac{A^2}{A_1^2}\right]^{1/2},$$

$$\Delta_\omega^{(2)} = (4\Delta^2-\omega^2)^{1/2}. \tag{7}$$

The plots are shown schematically in the figure, where

$$\frac{2\Delta-\omega_1}{\Delta} = \left(\frac{A}{A_1\sqrt{2}}\right)^{4/5}. \tag{8}$$

The lower dashed curve is unstable. The zero-order solution is also unstable when $\omega_1 < \omega < 2\Delta$. This region corresponds precisely to parametric resonance. Thus, the section AB of the upper curve is certainly realized. The maximum value of Δ_ω at $\omega = \omega_1$ is

$$\Delta_\omega = 2\Delta(A/A_1\sqrt{2})^{2/5}. \tag{9}$$

We note that it is not the quantity Δ_ω itself that depends on the field, but the region of instability of the zero-order solution. An essential feature of the phenomenon in question is that the correction to the gap varies with the frequency ω of the external field, whereas unually only the harmonics of Δ at

the frequencies 2ω, 4ω, etc. vary. This leads to a current harmonic 2ω instead of the usual 3ω:

$$j_{2\omega} = -\frac{\sigma}{c}(\Delta A)_{2\omega}(1{,}5 + 1{,}8i). \qquad (10)$$

The experiment can thus yield the following: in the frequency region $(2\Delta - \omega)/\Delta \sim (A/A_1)^{4/5}$ below the threshold there is a signal reflected from the film, with frequency 2ω and relative magnitude $j_{2\omega}/j_{\omega} \sim (A/A_1)^{2/5}$.

The foregoing calculation pertain to the case of a low temperature, $T \ll \Delta$. In the temperature region $T \sim \Delta$, all that occurs in addition to the increased damping γ is a renormalization of the coefficients.

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[1] A.A. Abrikosov, L.P. Gor'kov, and I.M. Khalatnikov, Zh. Eksp. Teor. Fiz. 35, 265 (1958) [Sov. Phys.-JETP 8, 182 (1959)].
[2] B.I. Ivlev, ZhETF Pis. Red. 15, 441 (1972) [JETP Lett. 15, 313 (1972)].
[3] L.D. Landau and E.M. Lifshitz, Mekhanika (Mechanics), Nauka, 1965 (Addison-Wesley, 1969].
[4] G.M. Eliashberg, Zh. Eksp. Teor. Fiz. 61, 1254 (1971) [Sov. Phys.-JETP 34, 668 (1972)].

TRANSITION OF HYDROGEN INTO THE METALLIC STATE IN A COMPRESSION WAVE INDUCED BY A LASER PULSE

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Theoretical calculations show (cf., e.g., [1 - 4]) that molecular solid hydrogen should go over into a metallic state at a pressure of several megabars. There have been several studies of the thermodynamic properties of the molecular [1, 2] and metallic [3, 4] phases, and the pressure of the transition has been estimated. It will be shown below that the pressures needed for the transition can be attained by irradiating a hydrogen target with a laser pulse of special waveform; the waveform of the pulse will be determined and the pulse parameters in the target will be estimated.

A high degree of compression at a relatively small temperature rise can be obtained by compressing the substance adiabatically and preventing the formation of strong shock waves. The possibility of realizing by laser irradiation a compression close to adiabatic, with a degree of compression $n/n_0 \sim 10^4$, to obtain a thermonuclear reaction with a positive yield, is discussed in [5]. In our case, the necessary degree of compression is of the order of 10, so that many complications considered in [5] are avoided; in particular, there is no need for spherically-symmetrical compression, and the radiation intensities obtainable from present-day lasers turn out to be sufficient.

To obtain adiabatic compression under laser irradiation, they used in [5] the fact that the thermal wave initiated by the laser pulse and propagating at subsonic velocity (the so-called wave of second kind, see [6]) acts like a piston and produces a compression wave ahead of it. The law governing the piston motion is determined from the requirement that the discontinuity in the compression wave occur not earlier than a certain specified instant of time. The complete determination of the optimal regime calls, of course, for numerical calculations similar to those in [5]. Such calculations are rather laborious and their actual accuracy is low, since the equation of state of hydrogen and