the frequencies 2ω, 4ω, etc. vary. This leads to a current harmonic 2ω instead of the usual 3ω:

$$j_{2\omega} = -\frac{\sigma}{c}(\Delta A)_{2\omega}(1{,}5 + 1{,}8\,i). \qquad (10)$$

The experiment can thus yield the following: in the frequency region $(2\Delta - \omega)/\Delta \sim (A/A_1)^{4/5}$ below the threshold there is a signal reflected from the film, with frequency 2ω and relative magnitude $j_{2\omega}/j_{\omega} \sim (A/A_1)^{2/5}$.

The foregoing calculation pertain to the case of a low temperature, $T \ll \Delta$. In the temperature region $T \sim \Delta$, all that occurs in addition to the increased damping γ is a renormalization of the coefficients.

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TRANSITION OF HYDROGEN INTO THE METALLIC STATE IN A COMPRESSION WAVE INDUCED BY A LASER PULSE

S.I. Anisimov
L.D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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Theoretical calculations show (cf., e.g., [1 - 4]) that molecular solid hydrogen should go over into a metallic state at a pressure of several megabars. There have been several studies of the thermodynamic properties of the molecular [1, 2] and metallic [3, 4] phases, and the pressure of the transition has been estimated. It will be shown below that the pressures needed for the transition can be attained by irradiating a hydrogen target with a laser pulse of special waveform; the waveform of the pulse will be determined and the pulse parameters in the target will be estimated.

A high degree of compression at a relatively small temperature rise can be obtained by compressing the substance adiabatically and preventing the formation of strong shock waves. The possibility of realizing by laser irradiation a compression close to adiabatic, with a degree of compression $n/n_0 \sim 10^4$, to obtain a thermonuclear reaction with a positive yield, is discussed in [5]. In our case, the necessary degree of compression is of the order of 10, so that many complications considered in [5] are avoided; in particular, there is no need for spherically-symmetrical compression, and the radiation intensities obtainable from present-day lasers turn out to be sufficient.

To obtain adiabatic compression under laser irradiation, they used in [5] the fact that the thermal wave initiated by the laser pulse and propagating at subsonic velocity (the so-called wave of second kind, see [6]) acts like a piston and produces a compression wave ahead of it. The law governing the piston motion is determined from the requirement that the discontinuity in the compression wave occur not earlier than a certain specified instant of time. The complete determination of the optimal regime calls, of course, for numerical calculations similar to those in [5]. Such calculations are rather laborious and their actual accuracy is low, since the equation of state of hydrogen and the absorption coefficient of light at high intensities are not accurately known. It is therefore necessary to make a simple analytical calculation, replacing the thermal wave by a piston and determining, from the condition that the flow be adiabatic, the equation of motion of the piston and the law governing the energy input. The calculation requires knowledge of the equation of the adiabat of molecular hydrogen. To obtain this equation we note that in the Debye model the entropy depends on the ratio T/θ, and therefore the equation of the adiabat is $T = \text{const}\cdot\theta(\rho)$ (at zero pressure we have $\theta_0 = 108°\text{K}$ and $\rho_0 = 0.089\ \text{g/cm}^3$). To avoid appreciable heating during the compression, it is necessary to choose the initial temperature much lower than θ_0. It is then easily seen that the thermal contribution to the pressure in the adiabatic process will remain small, $P_T \sim (T_0/\theta_0)^4$, and it is permissible to replace the adiabat by the cold-compression curve. We take the equation for the latter from [2] and approximate it, for ease in calculation, by a power-law function. In the pressure interval from several kbar to 5 Mbar, we can write the approximate formula in the form $P = a\rho^\gamma$, where $\gamma = 3$ and $a = 3.3 \times 10^6$ (the pressure is in bars and the density in g/cm^3).

It can be shown that the gasdynamic problem of inhomogeneous adiabatic compression by a piston has a simple self-similar solution. We shall not write it out here, but present the result of a calculation based on it. We consider planar compression, which is the most convenient from the point of view of setting up the experiment, and require that the shock wave not be produced before the piston travels a distance x_0. The motion of the piston is thus given by

$$x = x_0(1 + \tau - 2\sqrt{\tau}), \qquad \tau = 1 - \frac{c_0 t}{x_0},$$

where c_0 is the initial speed of sound[1]. Equating the power consumed in compression to the energy flow from the laser, we obtain the required waveform of the pulse

$$q(t) \approx \rho_0 c_0^3 \tau^{-2}(1 - \sqrt{\tau}).$$

The total energy (per unit area) consumed in compression by the instant of time t_m is

$$Q \approx \frac{1}{3}\rho_0 c_0^2 x_0 \tau_m^{-1}, \qquad \tau_m = 1 - \frac{c_0 t_m}{x_0}.$$

The value of τ_m, together with the maximum intensity and the necessary energy, is determined from the condition that the pressure in a layer of thickness δ exceed at the end of the compression the transition pressure p^*. This condition gives the connection between τ_m and x_0. Choosing x_0 from the condition that Q be a minimum, we get

$$\tau_m \approx 0.5\alpha^{-2}; \qquad x_0 \approx 2.5\,\alpha\delta; \qquad Q \approx 5\delta p^*,$$

where α is the degree of compression. The value of δ is determined by the experimental conditions. To obtain a numerical estimate, we put $\delta = 0.1$ mm and choose for the transition pressure the patently overestimated value $p^* = 5$ Mbar. We then obtain

$$q_{max} \approx 3\cdot 10^{13}\ \text{W/cm}^2; \quad Q \approx 2{,}5\cdot 10^4\ \text{J/cm}^2.$$

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[1] It can be shown that in the case of spherical and cylindrical compression, the laws of piston motion should be $r \sim \tau^{1/\gamma}$ and $r \sim \tau^{2/(3\gamma-1)}$, respectively.

To be able to regard the condition as one-dimensional, the dimension of the irradiated area should be of the order of x_0. Taking this into account, we find that the energy of the laser pulse (disregarding reflection and other losses) should be approximately 2 kJ.

The appearance of a metallic phase in compressed matter can be detected, in principle, by registering the reflection of the light from the front of the compression wave (through a layer of uncompressed matter).

A reliable estimate of the temperature entails certain difficulties, since our approach disregards completely the energy transfer by heat conduction. A lower-bound estimate follows obviously from the adiabatic equation. Using the $\theta(\rho)$ dependence from [2], we get

$$T \approx 300\, T_0 e^{-4.7/\alpha} .$$

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