[1] D.R. Bates and R. McCarroll, Proc. Roy. Soc. A245, 175 (1958)
[2] B.M. Smirnov, Opt. Spektr. 17, 504 (1964).
[3] D. Coffey, Jr., D.C. Lorents, and F.T. Smith, Phys. Rev. 187, 201 (1969).
[4] D.C. Lorents and G.M. Conklin, J. of Phys. B, Atomic and Molecular Physics 5, 950 (1972).
[5] R. Francois, D. Dhuicq, and M. Barat, J. of Phys. B, Atomic and Molecular Physics 5, 963 (1972).
[6] R.McCarroll and R.D. Piacentini, J. of Phys. B, Atomic and Molecular Physics 5, 973 (1972)
[7] A.A. Latypov and A.A. Shaporenko, Zh. Tekh. Fiz. 42, 147 (1972) [Sov. Phys.-Tech. Phys. 17, 114 (1972)].

THE SHUBNIKOV - DE HAAS EFFECT IN THIN CONDUCTORS

V.G. Peschanskii and V.V. Sinolitskii
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
Submitted 15 September 1972
ZhETF Pis. Red. 16, No. 8, 484 - 487 (20 October 1972)

In the quasiclassical region of magnetic fields, when the distance $\hbar\Omega$ between the quantized energy levels of the conduction electrons is much less than the Fermi energy ε_0 but is larger than their width $\hbar/\tau$, the resistivity ρ of bulky conductors oscillates as a function of the reciprocal magnetic field, and the extremal areas of the sections of the Fermi surface can be determined from the periods of the oscillations [1, 2]. In sufficiently thin plates and wires, the electron orbit corresponding to the maximum area S_{max} of the section of the Fermi surface cannot be contained in the cross section of the conductor, and at magnetic field values H at which the diameter of such an orbit is equal to the thickness d of the conductor, i.e.,

$$\frac{cD_p}{eH} = d, \qquad (1)$$

the Shubnikov - de Haas oscillations with period $\Delta(1/H) = 2\pi\hbar e/cS_{max}$ should vanish, in analogy with the cutoff of cyclotron resonance frequencies [3]. Here e is the electron charge, Ω is the frequency of its revolution in the magnetic field, τ is the free-path time, $\hbar$ is Planck's constant, c is the speed of light, and D_p is the parameter of the cross section of the Fermi surface.

Just as in the case of cyclotron resonance, when a new resonant frequency that depends on the sample thickness appears in place of the cut-off frequency [4], one should expect at $H < (cD_p/ed)$ the appearance of a new oscillatory dependence of the resistance on the magnetic field and on the conductor thickness. The period of these oscillations is connected with the characteristics of the electrons that do not collide with the sample boundary and for which the cross section of the Fermi surface is maximal. Since the diameter of the orbit of such electrons satisfies relation (1), it is possible to determine from the periods of the $\rho(H)$ oscillations, at different values of d, the connection between the area $S(\varepsilon_0, p_z)$ and the diameter $D(p_z)$ of the section of the Fermi surface for all values of the electron monentum projection p_z on the magnetic-field direction.

The character of the reflection of the carriers from the sample boundary is immaterial for this oscillatory effect. We shall assume that the electrons are diffusely scattered upon collision with the surface of the conductor, and that it suffices to take into account their contribution to the electron-current density

$$j = \mathrm{Sp}\, e\, \hat{v}\, \hat{f} \qquad (2)$$

classically by using the Boltzmann kinetic equation for the carrier distribution function (cf., e.g., [5]). For electrons that do not collide with the surface of the sample it is necessary to solve, in an approximation linear in the weak electric field $\vec{E}$, the quantum-kinetic equation for the density matrix $\hat{f} = \hat{f}_0 + \hat{f}_1$:

$$\frac{i}{\hbar}[\hat{\epsilon}_0, \hat{f}_1] + \hat{W}\hat{f} = \frac{i}{\hbar}[e\mathbf{E}\hat{\mathbf{r}}, \hat{f}_0], \qquad (3)$$

where $\hat{\epsilon}_0$ and $\hat{f}_0$ are the Hamiltonian operator and density matrix of the conduction electrons in the absence of an electric field, and $\hat{\vec{v}}$ and $\hat{\vec{r}}$ are the electron velocity and coordinate operators.

If the carrier dispersion law $\epsilon = \epsilon(\vec{p})$ is anisotropic, then the component of the electric field in the plane of the sample cross section can be different from zero even when $\vec{j} \parallel \vec{H}$; this component should be determined from the conductor electroneutrality condition

$$\mathrm{Sp}\, e\hat{f}_1 = 0. \qquad (4)$$

In the quantum analog of the collision integral $\hat{W}\hat{f}$ is it necessary in this case to take into account terms proportional to $\vec{E}$, just as in the investigation of transverse galvanomagnetic effects [6, 7]. Assuming that the scattering of the electrons inside the conductor is elastic[1)], we can obtain, in analogy with the procedure used by Kosevich and Andreev [7], as explicit expression for the operator $\hat{W}\hat{f}$ and solve Eq. (3) by successive approximations in terms of the small parameter $1/\Omega\tau \ll 1$.

The remaining calculation of the resistance of a thin plate or wire in a magnetic field parallel to its surface entails no difficulty. Using the Poisson formula and the area quantization rule

$$S(\epsilon, p_z) = 2\pi\hbar(n + \gamma)eH/c, \qquad 0 \leqslant \gamma < 1,$$

we can replace the summation over integer non-negative n in (2) by integration with respect to the energy ϵ. The small quantum corrections to the electric-current density are calculated by the stationary-phase method. If the area $S(\epsilon_0, p_z)$ of the section of the Fermi surface for electrons whose orbits are not tangent to the boundaries of the conductor is a monotonic function of p_z, then the quantum increment to the resistance $\rho(H, d)$ is determined only by the small region near the limiting point of the Fermi surface and near values of p_z satisfying Eq. (1). The contribution of the latter has an oscillatory character.

Omitting the intermediate calculations, we present the final results for the oscillating part of the longitudinal resistance of a plate

$$\frac{\Delta\rho_{osc}}{\rho_0} \approx \frac{\lambda^2 \ell}{d^3}\left(1 + \frac{\ell d}{r^2}\right)^{-2} \ln\frac{\ell}{d} \cos\left(a\frac{eHd^2}{ch} - 2\pi\gamma\right) \qquad (5)$$

[1)] The Knudsen case (electron mean free path $\ell = v\tau$ much larger than the sample thickness) is realizable only at low temperatures when the electrons are scattered mainly by impurities.

and a wire

$$\frac{\Delta\rho_{osc}}{\rho_o} \approx \frac{\lambda\!\!\!^{-3} r \ell}{d^5}\left(1 + \frac{\ell d}{r^2}\right)^{-2} \sin\left(\alpha \frac{eHd^2}{c\hbar} - 2\pi\gamma\right), \tag{6}$$

and also of transverse resistance of the plate when the magnetic field is perpendicular to the current but lies in the plane of the plate.

$$\frac{\Delta\rho_{osc}}{\rho_o} \approx \frac{\lambda\!\!\!^{-} r}{\ell d} \ln\frac{\ell}{d} \cos\left(\alpha \frac{eHd^2}{c\hbar} - 2\pi\gamma\right); \tag{7}$$

$$\rho_o \equiv \rho(0, d); \quad \lambda\!\!\!^{-} \equiv \hbar/D_p^{max}; \quad r \equiv cD_p^{max}/eH; \quad d < r << \ell.$$

An experimental investigation of these oscillations makes it possible to determine the dimensionless factor α that connects the area of the section of the Fermi surface with the square of its diameter

$$S(\epsilon_o, p_z) = \alpha D^2(p_z),$$

for those values of p_z which are the roots of the equation $D(p_z) = eHd/c$, i.e., of Eq. (1). Since α depends on Hd (only for an isotropic dispersion law do we have $\alpha = \pi/4$ at any p_z), it is advantageous to plot a family of $\rho_{osc}(d)$ curves at Hd = const and obtain with their aid detailed information concerning the non-extremal sections of the Fermi surface.

The foregoing formulas are valid only for single-crystal samples. For polycrystalline conductors whose thickness is of the order of the dimensions of the crystallite, the oscillation amplitude is smaller than in (5) - (7), owing to the averaging over the random orientations of the crystallites, and the period of the oscillations is determined by the area of the Fermi-surface section that is farthest from the limiting point and whose diameter satisfies relation (1). If the investigated cylinders are not strictly cylindrical (plate with variable thickness or wire with variable cross section), then d in (1) must be replaced by d_{max}.

[1] L.W. Schubnikov and W. de Haas, Leid. Comm. 207, 210 (1930).
[2] I.M. Lifshitz and A.M. Kosevich, Zh. Eksp. Teor. Fiz. 33, 88 (1957) [Sov. Phys.-JETP 6, 67 (1958)].
[3] M.S. Khaikin, ibid. 41, 1773 (1961) [14, 1260 (1962)].
[4] M.A. Lur'e and V.G. Peschanskii, Abstracts of Papers of 17th All-union Conference on Low Temperature Physics, Donetsk, 1972.
[5] V.G. Peschanskii and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 55, 1980 (1969) [Sov. Phys.-JETP 28, 1045 (1970)].
[6] E.N. Adams and T.D. Holstein, J. Phys. Chem. Solids 10, 254 (1959).
[7] A.M. Kosevich and V.V. Andreev, Zh. Eksp. Teor. Fiz. 38, 882 (1960) [Sov. Phys.-JETP 11, 637 (1960)].