DAMPING OF A SOLITON IN A PLASMA

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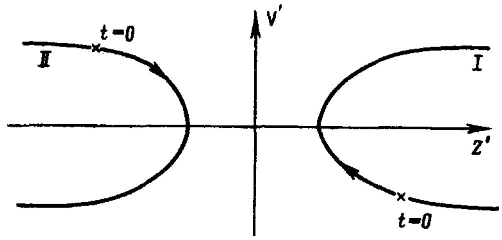
We investigate here adiabatic damping of a soliton as a result of absorption of its energy by resonant plasma particles. According to [1], the soliton energy dissipation in such an interaction is one of the mechanisms that lead to formation of a collisionless shock wave.

The dissipation is connected mainly with particles reflected from the "hump" of the potential energy in the soliton. The phase trajectories of these particles, plotted in the wave coordinates $v' = v - v_{ph}$ and $z' = z - \int_0^t v_{ph} dt$, are shown in the figure. The particles move in the accelerating phase of the field on trajectories I, and in the decelerating phase on trajectories II. When $\partial f_0/\partial v_{ph} < 0$ ($f_0(v)$ is the equilibrium distribution function of the plasma) there are more particles in the accelerating phase, and the soliton is damped. The nonlinear damping stabilization due to the phase oscillations of the resonant particles [2, 3] does not exist for the reflected particles, and the amplitude of the soliton attenuates to zero as a result of interaction with such particles. When $\partial f_0/\partial v_{ph} > 0$, i.e., in the presence of a beam in the plasma, the interaction with the reflected particles leads to an intensification of the soliton. The amplitude of the potential in the soliton increases in this case to values $e|\phi|^{max} \sim m_e v_{ph}(v_0 - v_{ph})$ (v_0 is the beam velocity) much higher than for a monochromatic wave.

We consider first the damping of a high-frequency Langmuir soliton. Such a soliton is produced in a magnetized plasma ($\omega_{He} >> \omega_{pe}$, where ω_{He} and ω_{pe} are the electron cyclotron and plasma frequencies, respectively) if the wave propagates at an angle to the magnetic field ($k_\perp \neq 0$) with a phase velocity $v_{ph} > (\omega_{pe}/k_\perp)$ [4, 5]. Assuming $v_{ph} = (\omega_{pe}/k_\perp)(1 + \delta)$, $\delta << 1$, we confine ourselves to a soliton of small amplitude $(e|\phi|^{max}/m_e v_{ph}^2) << 1$. In this approximation the equation for the potential of the Langmuir wave $\phi(t, x, z')$ (for convenience, the prime will henceforth be omitted) takes the form

$$\frac{\partial^2\phi}{\partial x^2} + k_\perp^2\phi = -\frac{\partial^2\phi}{\partial z^2} + \frac{\omega_{pe}^2}{v_{ph}^2}\delta\phi + \frac{3}{2}\frac{\omega_{pe}^2}{v_{ph}^2}\frac{e\phi^2}{m_e v_{ph}^2} - 2\frac{\omega_{pe}^2}{v_{ph}^3}\int_{-\infty}^{z}\frac{\partial\phi}{\partial t}dz' + 4\pi e n_{res}. \quad (1)$$

The last two terms in (1) describe the slow variation of the soliton parameters as it interacts with the resonant particles, and n_{res} is the perturbation of the density of the resonant electrons in the wave. All the small quantities $\lesssim\delta$ have been transferred to the right-hand side of Eq. (1). We seek a solution $\phi = \phi^{(0)} + \phi^{(1)} + \ldots$ of this equation by expanding with respect to the parameter δ. In the zeroth



approximation we have

$$\phi^{(0)} = \psi(z, t) \cos k_\perp x . \tag{2}$$

We have chosen a solution satisfying the condition $\phi(-x) = \phi(x)$. The spectrum of $k_\perp$ is determined from the boundary conditions at $x = \pm a$. Thus, if the wave propagates in a waveguide with conducting walls we have $\phi(\pm a) = 0$, $k_\perp a = \pi(2n + 1)/2$, $n = 0, 1, \ldots$ We shall consider henceforth the first mode with $n = 0$. The equation for $\psi(z, t)$, as usual, is obtained from the condition of orthogonality of the right-hand side of (1) to $\phi^{(0)}(x)$:

$$\frac{\partial^2 \psi}{\partial z^2} - \frac{\omega_{pe}^2}{v_{ph}^2} \delta \psi - \frac{4}{\pi} \frac{\omega_{pe}^2}{v_{ph}^2} \frac{e \psi^2}{m_e v_{ph}^2} + \frac{2\omega_{pe}^2}{v_{ph}^3} \int_{-\infty}^{z} \frac{\partial \psi}{dt} dz' - \frac{4\pi e}{a} \int_{-a}^{a} n_{res} \cos k_\perp x \, dx = 0. \tag{3}$$

If the condition $|\partial\psi/\partial t| \ll \delta v_{ph} |\partial\psi/\partial z|$ is satisfied, Eq. (3) has a solution in the form of a soliton

$$\frac{e\psi(z,t)}{m_e v_{ph}^2} = -\alpha(t)\, \mathrm{ch}^{-2} \frac{z}{\Delta(t)}, \quad \Delta = \sqrt{\frac{3\pi}{2\alpha}} \frac{v_{ph}}{\omega_{pe}}, \quad \alpha = \frac{3\pi}{8}\delta. \tag{4}$$

The function $\alpha(t)$ is determined from the equation

$$\int_{-\infty}^{\infty} dz \frac{\partial \psi}{\partial z} \int_{-\infty}^{z} dz' \frac{\partial \psi}{\partial t} = \frac{2\pi e v_{ph}^3}{\omega_{pe}^2 a} \int_{-\infty}^{\infty} dz \frac{\partial \psi}{\partial z} \int_{-a}^{a} dx \cos k_\perp x \times \int dv\, f_{res}(t, z, x, v). \tag{5}$$

$f_{res}(t, x, z, v)$ is the distribution function of the resonant particles. We transform the right-hand side of Eq. (5) by using the Liouville phase space conservation theorem $dzdv - dz_0 dv_0$[1] and the condition that the distribution function be constant on the particle trajectories, $f(t, x, z, v) = f_0(v_0 + v_{ph})$. The equations for the resonant-particle trajectories in the soliton field take the form $u = u_0 \pm [2\mathcal{E}/m_e]^{1/2} t/\Delta$, with the + and - signs corresponding to particles with $v_0 > 0$ and $v_0 < 0$, respectively. u is connected with z by the relation

$$\mathrm{sh}\frac{z}{\Delta} = \mu\, \mathrm{sh}\, u, \quad \mu^2 = \frac{\mathcal{E} + e\phi_0}{\mathcal{E}}, \quad \phi_0 = \phi(z = 0) = -\alpha \frac{m v_{ph}^2}{e} \cos k_\perp x$$

for particles that pass through with energy $\mathcal{E} > -e\phi_0$ and

$$\mathrm{sh}\frac{z}{\Delta} = \tilde{\mu}\, \mathrm{ch}\, u, \quad \tilde{\mu}^2 = -\frac{\mathcal{E} + e\phi_0}{\mathcal{E}}$$

for reflected particles, i.e., for $\mathcal{E} < -e\phi_0$. From (5) we obtain the following equation for $\alpha(t)$:

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[1] z_0 and v_0 are the initial coordinates of the particle on the phase trajectory that passes at the instant of time t through the point z, v.

$$\frac{da}{dt} = -4\sqrt{\frac{2}{3\pi^3}}\,\frac{\omega_{pe}}{m_e n_o}\,\frac{\partial f_o}{\partial v_{ph}}\,\alpha^{1/2}\int_0^{\pi/2} d\xi_o \cos\xi_o \left\{\int_{-e\phi_o}^{\infty} d\mathcal{E}\,\mu^2 \times\right.$$

$$\times \sum_{j=\pm 1} j\int_0^{\infty} du_o \frac{\operatorname{ch} u_o}{(1+\mu^2 \operatorname{sh}^2 u_o)^{1/2}}\,\frac{\operatorname{sh}\left(u_o + j\sqrt{\frac{2\mathcal{E}}{m_e}}\frac{t}{\Delta}\right)}{\left[1+\mu^2\operatorname{sh}^2\left(u_o + j\sqrt{\frac{2\mathcal{E}}{m_e}}\frac{t}{\Delta}\right)\right]^{3/2}} +$$

$$\left. + \int_0^{-e\phi_o} d\mathcal{E}\,\tilde{\mu}^2 \sum_{j=\pm 1} j\int_0^{\infty} du_o \frac{\operatorname{sh} u_o}{(1+\tilde{\mu}^2 \operatorname{ch}^2 u_o)^{1/2}}\,\frac{\operatorname{ch}\left(u_o + j\sqrt{\frac{2\mathcal{E}}{m_e}}\frac{t}{\Delta}\right)}{\left[1+\tilde{\mu}^2\operatorname{ch}^2\left(u_o + j\sqrt{\frac{2\mathcal{E}}{m_e}}\frac{t}{\Delta}\right)\right]^{3/2}}\right\}. \qquad (6)$$

To obtain this equation, we have represented the equilibrium distribution function of the resonant particles in the form $f_0(v_{ph} + v_0) = f_0(v_{ph}) + v_0 \partial f_0/\partial v_{ph}$ and have eliminated the integral with respect to $z_0 < 0$ with the aid of the condition $z(-z_0, -v_0, t) = -z(z_0, v_0, t)$. At times t that are large in comparison with the time of flight of the particles through the soliton, $t \gg (\Delta/v_{ph})(1/\sqrt{\alpha})$, only the contribution of the reflected particles with $v_0 < 0$ is significant in (6). We then get from (6) the simple equation

$$\frac{da}{dt} = \sqrt{\frac{2}{3\pi}}\,\frac{\omega_{pe}}{n_o}\,v_{ph}^2\frac{\partial f_o}{\partial v_{ph}}\,a^{3/2}, \qquad (7)$$

the solution of which

$$a(t) = \frac{a(0)}{\left(1 + \sqrt{\frac{a(0)}{6\pi}}\,\gamma_L t\right)^2}, \quad \gamma_L = -\frac{\omega_{pe}}{n_o}v_{ph}^2\frac{\partial f_o}{\partial v_{ph}} \qquad (8)$$

describes the damping of a Langmuir soliton as a result of interaction with reflected particles.

In a non-isothermal plasma ($T_e \gg T_i$) there can exist a low-frequency ion-acoustic soliton [6]. In such a soliton, unlike the one considered above, we have $\phi > 0$ and the damping of the soliton is due to the electrons reflected from the "hump" of the potential. For a soliton of sufficiently low amplitude $\alpha_s = e\phi^{max}/m_i v_{ph}^2 \ll 1$, the damping decrement $\gamma_i(t)$ due to the reflected particles is calculated in analogy with the procedure used above for the Langmuir soliton.

The resonant electrons are captured in a potential well produced by the ion-acoustic soliton. The phase "mixing" of the captured particles causes the electronic damping decrement to decrease within a time $t \gg 1/\omega_{pe}\alpha_s$, i.e., sufficiently rapidly, to the value

$$\gamma_e^{\infty} \approx \gamma_i\sqrt{\alpha_s} \ll \gamma_i .$$

The presence of γ_e^{∞} is connected with the adiabatic realignment of the electronic trajectories in the field of a soliton with a time-varying amplitude. The electrons thus do not influence the damping of the soliton significantly.

Allowance for the damping due to the reflected ions leads, for times $t \gg 1/\omega_{pi}\alpha_s$, to the following formula for the amplitude of the ion-acoustic soliton:

$$a_s(t) = \frac{a_s(0)}{\left(1 + \sqrt{\frac{a_s(0)}{24}}\gamma_s t\right)^2}, \quad \gamma_s = -\frac{\omega_{pi}}{n_0} v_{ph}^2 \frac{\partial f_{oi}}{\partial v_{ph}} \tag{9}$$

$f_{0i}(v)$ is the equilibrium distribution of the ions, and $\omega_{pi} = (m_e/m_i)^{1/2}\omega_{pe}$.

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HADRONIC SYMMETRIES AND GAUGE THEORIES OF WEAK AND ELECTROMAGNETIC INTERACTION

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1. Unified renormalizable theories of weak and electromagnetic interactions with spontaneously-violated gauge invariance have been extensively discussed of late [1 - 7]. These theories make it possible to describe in a unified manner both weak and electromagnetic interaction, unifying the photon and the $W^{\pm}$ bosons (usually with one more neutral vector meson) as gauge fields.

The proposed models [1 - 5] make it possible to describe purely leptonic processes, but when they are generalized to include hadrons, many difficulties are raised by the observed SU(3) and SU(6) symmetries of the strong interactions. Thus, 12 quarks are needed in [2] to construct the known SU(3) baryon multiplets, and seven and eight quarks are needed in the models of [3, 4]. In addition, in view of the integer charge of the quarks in the models of [3 - 5], SU(6) symmetry is lost for baryons.

2. We propose in this article a renormalizable theory of weak and electromagnetic interactions with spontaneously-violated SU(2) × U(1) gauge symmetry, including the usual SU(3) quarks with fractional charge, and allowing us to preserve the observed hadronic symmetries.

We postulate the usual SU(3) triplet quark (p, n, λ), two new quarks p' and q with charges +2/3 and -1/3, and two new heavy neutron leptons, electronic (E) and muonic (M), in addition to e, ν_e, μ, and ν_μ. The strong SU(5) interactions are invariant, and the observed SU(3) symmetry is the low-energy limit of the global SU(5) symmetry if the quarks p' and q are much heavier than the triplet (p, n, λ).

The gauge vector fields, the SU(2) triplet (W^+, S^0, W^-) and the singlet B^0, are introduced in accordance with