

in the  $\sigma_m^i$  meson system, then this could lead, as a result of the vacuum mean values  $\langle \sigma_k^i \rangle$ , to terms of the type of the bare masses used in (9). On the other hand, in the absence of an interaction between the hadrons and the  $\sigma_k^i$  and  $\pi_k^i$  fields, the hadrons would be massless and we would then have an exact  $SU(5) \times SU(5)$  symmetry of strong interactions. In this case, possibly, we might be able to fit the PCAC hypothesis into the framework of this scheme, in analogy with the  $\sigma$  model of Gell-Mann and Levy [10].

In conclusion, I am deeply grateful to B.L. Ioffe for a valuable discussion and remarks, which have stimulated to a considerable degree the publication of this paper.

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#### DISTRIBUTION WITH RESPECT TO THE TRANSVERSE MOMENTUM IN THE MULTIPERIPHERAL MODEL

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Submitted 12 September 1972

ZhETF Pis. Red. 16, No. 8, 495 - 499 (20 October 1972)

In an earlier paper [1, 2] we proposed a multiperipheral model that ensures a constant cross section of particle interaction at high energies. We obtain here the distribution with respect to the transverse momenta of the muons produced in this model in collisions between two particles, and show that it agrees fairly well with experiment [3 - 6]. Let us recall the main features of the model.

To obtain a non-decreasing total cross section at reasonable values of the coupling constants it is necessary to take into account the dependence of the amplitudes on the pair energies [2] and the possibility of emission of different types of particles. Therefore, besides pion exchange, we take into account exchange of the  $\rho$  and  $\omega$  trajectories and allow the production of  $\pi$ ,  $\rho$ ,  $\omega$ ,  $f$ , and  $A_2$  mesons.

Then the matrix element shown in Fig. 1a takes the form

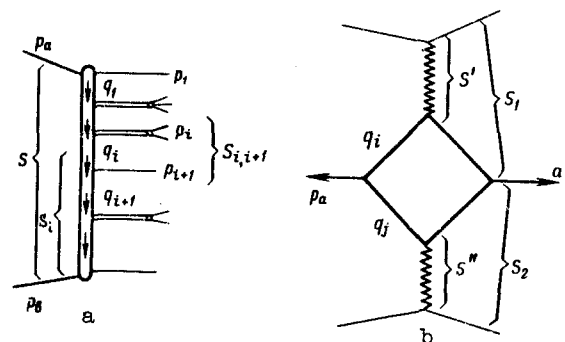


Fig. 1

$$M = \prod_i \gamma_{i,i+1}^a(q_i^2, q_{i+1}^2) \beta_{i+1}(q_{i+1}^2) x_i^{\alpha_{i+1}(q_{i+1}^2)} e^{R^2 q_{i+1}^2} \quad (1)$$

Here  $\gamma_{i,i+1}^a(q_i^2, q_{i+1}^2)$  is the vertex for the emission of particle "a,"  $\beta(q^2)$  is the pion propagator or the signature of the  $\rho$  or  $\omega$  trajectory,  $x_i + S_{i+1}/S_i$ , and  $x_i^\alpha$  results from the reggeon propagator  $S_{i,i+1}$ ; the vertices  $\gamma$  were estimated from the dual models [7]. We chose the radius  $R$  such as to obtain a constant cross section. The required value of  $R$  turned out to be close to the estimates based on the dual amplitudes [7]. To find the distribution with respect to the perpendicular momenta of the emitted particles, let us examine the diagram 1b. The inclusive cross section for particle production in the pionization region (from the middle of the ladder) depends only on  $p_\perp$  and is given by

$$f^a(p_\perp) = \frac{E_a d\sigma}{d^3p_a} = \sum_{i,j} \iint dq_i^2 dq_j^2 \int_0^1 \int_0^1 dx dy |\gamma_{i,i}^a|^2 \sigma_i(q_i^2) \sigma_j(q_j^2) \times$$

$$\times \frac{x^{1-2\alpha_i} y^{1-2\alpha_j} \kappa^{2-2\alpha_i}}{(1-x)(1-y) 2^8 \pi^7} \times$$

$$\times |\beta_i(q_i^2) \beta_j(q_j^2)|^2 T(-q_i^2 - x\kappa, -q_{i+1}^2 - y\kappa, |p_\perp^2|), \quad (2)$$

where

$$\kappa = \frac{m_\sigma^2 + |p_\perp^2|}{(1-x)(1-y)}; \quad x = S'/S_1; \quad y = S''/S_2;$$

$$T(a, b, c) = \theta(\lambda) \lambda^{-1/2}; \quad \lambda = 4ab - (a+b-c)^2$$

$\sigma_i(q_i^2)$  is the total cross section for the scattering of a particle of type  $i$  from the target. This is precisely the eigenfunction obtained in the construction of the model of [1].

Expression (2) differs from those in [8, 9] by a factor  $x^{-2\alpha_i} y^{-2\alpha_j}$  corresponding to the reggeon propagators and to summations of the particles of types  $i$  and  $j$  exchanged in the  $t$ -channel. The appearance of the factor  $\kappa^{-2\alpha_i}$  is connected with the definition given in [1] for the vertex  $\gamma$ .

By numerically integrating (2) we obtain  $f^a(p_\perp)$  for each type of emitted particle ( $\pi, \rho, \omega, f, A_2$ ). Since the radius  $R \sim 1 \text{ GeV}^{-1}$  is not large, and the smallness of the pion mass does not matter in the model of [1], the average transverse momenta of the resonances ( $\rho, \omega, f, A_2$ ) turn out to be quite large,  $\langle p_\perp^2 \rangle_\rho \gtrsim 0.4 \text{ GeV}^2$  (see Fig. 2), whereas the experimentally determined perpendicular momenta of pions are of the order of  $(2-3)m_\pi$ .

The small value of the transverse pion momentum can be easily understood if it is assumed that an appreciable fraction of the pions is the result of resonance decay [9, 10]. Indeed, if each pion carries away half of the  $\rho$  momentum (as would be the case if  $m_\pi = m_\rho/2$ ), then  $\langle p_\perp^2 \rangle_\pi = (1/4) \langle p_\perp^2 \rangle_\rho$ . A more exact form of the inclusive cross section of the pions produced in the  $\rho$  decay is

$$f_{\rho}^{\pi}(k_{\perp}) = \frac{2}{\pi} \int d\mathbf{p}_{\perp}^2 \frac{3}{2} \cos^2 \theta d\cos \theta f_{\rho}^{\rho}(\mathbf{p}_{\perp}) T(|\mathbf{p}_{\perp}^2| X^2, q^2 \sin^2 \theta, |k_{\perp}^2|). \quad (3)$$

Here  $X$  is the fraction of the  $\rho$  momentum carried away by the pion,  $X = m_{\pi}(1 + v \cos \theta) m_{\rho}^{-1} (1 - v^2)^{-1/2}$ ,  $v$  and  $\vec{q}$  are the pion velocity and momentum in the  $\rho$  rest system, and  $\theta$  is the angle between  $q$  and the direction of the colliding particles. The factor 2 in front of the integral corresponds to the decay of  $\rho$  into two pions. Figure 2 shows  $f_{\rho}^{\rho}(\mathbf{p}_{\perp})$  and  $f_{\rho}^{\pi}(k_{\perp})$  for the simple model of [2], in which only  $\rho$  mesons are emitted and exchanged. (We used the following parametrization:  $\gamma = 16\pi \cdot 6.2 \text{ GeV}^2$ ,  $\beta(q^2) = 1/(q^2 - m^2)$ ,  $\alpha(q^2) = 1/2$ ,  $R = 0$ . This yields  $\sigma = \text{const } (S) = 55 \text{ mb}$ ,  $N_{\pi} = 1 \ln S$ .) The pion distribution is much more peaked than the  $\rho$  distribution. The peak at small  $k_{\perp}$  is produced by the pions traveling backwards in the decay ( $\cos \theta = -1$ ). These pions have small  $X \sim 0.1$ , and their transverse momentum is determined mainly by the  $\rho$ -meson decay energy,  $\vec{k}_{\perp} = \vec{q}_{\perp} + X\vec{p}_{\perp} \sim \vec{q}_{\perp}$ . Therefore, at  $k_{\perp\pi} < 0.1 \text{ GeV}$  the slope of the curve is smaller than in the central part. At large  $k_{\perp}$ , the slope again decreases and the curve duplicates the  $\rho$ -meson distribution. This region corresponds to pions traveling forward ( $\cos \theta \sim 1$ ) and carrying away almost the entire momentum of  $\rho$  ( $k_{\perp\pi} \sim p_{\perp\rho}$ ). Similarly, the distribution of  $\omega$ ,  $f$ , and  $A_2$  are recalculated in terms of the  $\pi$  distribution. The final results are shown in Fig. 3. For comparison, the figure shows the experimental data on the inclusive cross sections of  $\pi^{\pm}$  mesons produced in  $pp$  collisions at  $89^\circ$  [3] in the c.m.s., i.e., precisely of those pions which are emitted from the center of the ladder, when formula (2) holds. The curves duplicate Fig. 2 in their behavior. In the region  $p_{\perp} \sim 0.1 - 0.9 \text{ GeV}$ , all the variants are in fair agreement with one another and with experiment, and are well described by the function  $\exp(-bp_{\perp})$ , where  $b \approx 5.5 \text{ GeV}^{-1}$ . At  $p_{\perp} < 0.1 \text{ GeV}$ , a decrease of the slope is observed throughout.

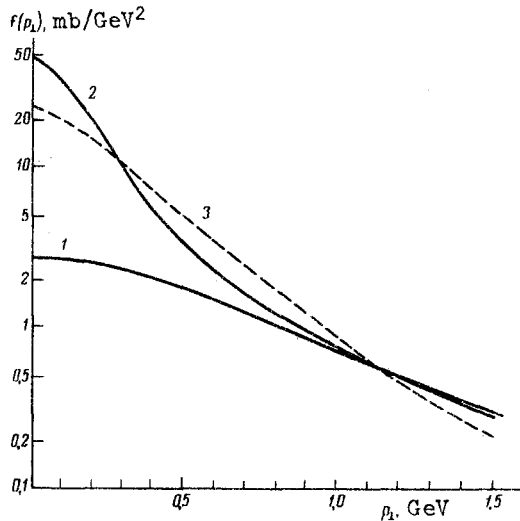


Fig. 2

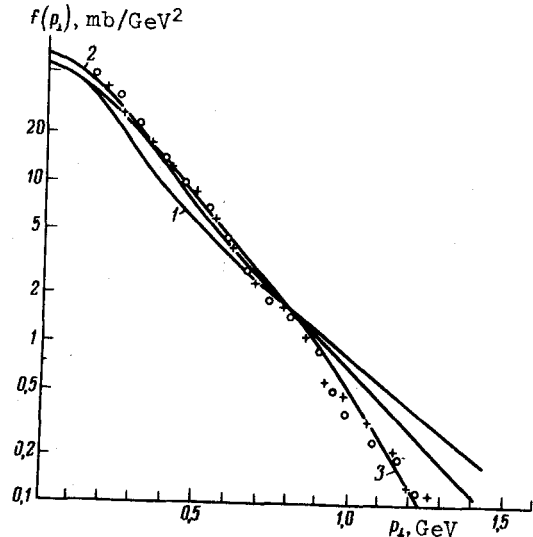


Fig. 3

Fig. 2. Inclusive cross section of  $\rho$  mesons (curve 1) and of the pions produced in their decay (curve 2) in the model of [2]. The dashed curve 3 shows the pion distribution in the case of isotropic  $\rho$  decay.

Fig. 3. Inclusive cross section of pions for the three variants (curves 1, 2, and 3) of [1]. The circles and crosses mark the experimental data [3] on the production of  $\pi^-$  and  $\pi^+$  mesons at  $89^\circ$  and  $E_{\text{c.m.s.}} = 30.4 \text{ GeV}$ .

A behavior of the same type (but at  $p_{\perp} < 0.2$  GeV) was observed at CERN [6] at lower energies. At large  $p_{\perp}$ , the variants differ from one another and depend on the details of the parametrization. A change of the parametrization in this region has hardly any effect on the magnitude and behavior of the total cross sections. Therefore new data at large  $p_{\perp}$  can be used to refine the model without changing the earlier results [1]. We note in conclusion that the multiperipheral model describes well the inclusive cross section of pions not only qualitatively but also quantitatively.

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#### MUON DECAY AND EQUIVALENCE PRINCIPLE

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Submitted 12 September 1972

*ZhETF Pis. Red.* 16, No. 8, 499 - 501 (20 October 1972)

The known formula for the interval

$$\Delta r = \int ds = \int (g_{\alpha\beta} dx^{\alpha} dx^{\beta})^{1/2} \quad (1)$$

makes it possible to calculate the connection between the invariant time and the observer's time  $t$ .

Example: In a gravitational field with potential  $\phi$ , this connection, accurate to terms in  $1/c^2$ , is given by

$$\Delta r \approx \int \left( 1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2} \right) dt = \int dt - \frac{1}{mc^2} \int L dt, \quad (2)$$

where  $L$  is the nonrelativistic Lagrange function.

How should the proper time be calculated for a particle (say a muon) in a magnetic field? Is this time also determined by an integral of the Lagrange function, i.e., by the action? It would seem that the action is the only quantity that can enter in the expression for the proper time, at least in the approximation under consideration. By virtue of the equivalence principle, it would seem that it is impossible in this case to distinguish between a potential in a gravitational and an electromagnetic field. Therefore the decay of a meson moving on a circular orbit in a cyclotron should be calculated by means of a formula similar to that by the proper time measured with a clock on an earth satellite.

Substituting in (2)