$$L = \frac{mv^2}{2} + \frac{e}{c} vA \quad (3)$$

and integrating over a circular orbit, assuming the magnetic field B to be constant, we obtain

$$\frac{1}{mc^2} \oint L\, dt = \frac{v^2}{2c^2} \int dt + \frac{e}{mc^3} \Phi, \quad (4)$$

where Φ is the magnetic flux enclosed by the particle orbit and ∫dt is the time of one revolution.

With the aid of (2) we obtain

$$\Delta\tau \cong \Delta t \left[1 - \left(1 + \frac{\Phi}{\pi R^2 B}\right) \frac{v^2}{2c^2}\right]. \quad (5)$$

For a meson moving in a straight line we have in this approximation

$$\Delta\tau \cong \left(1 - \frac{v^2}{2c^2}\right) \Delta t. \quad (6)$$

If the particle revolves in a homogeneous field (as in a cyclotron), then the coefficient in the round brackets of (5) is equal to 2. If the field differs from zero only in a narrow band (as in a synchrotron), then formula (6) holds true.

It would be useful to verify formula (5) experimentally.

Formula (5) can be regarded as a formula for the lifetime corrected for the external field. The correction is proportional to the zeroth Fourier component of the interaction, i.e., it is proportional to the action. The gravitational field can be taken into account in this approximation also in the field-theory formalism, assuming the metric to be plane. In such an approach, the difference between the gravitational and magnetic field seems to vanish in our approximation.

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MEASUREMENT OF THE ENERGY OF RELATIVISTIC PARTICLES WITH THE AID OF AN ONDULATOR IN AN OPTICALLY TRANSPARENT MEDIUM

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Measurement of the total energy W of an individual particle in the ultra-relativistic region $Mc^2/W = (1 - \beta^2)^{1/2} \ll 1$ encounters certain difficulties. Thus, Cerenkov counters are not very sensitive to changes of W in this region. Indeed, the radiation occurs at an angle $\theta_0 = \cos^{-1}(1/\beta n(\omega))$ to the particle velocity v, and the energy radiated over a path ℓ is

$$S_0 = \frac{e^2 \ell}{c^2} \int_{\beta n \geq 1} \sin^2 \theta_0\, \omega\, d\omega = \frac{e^2 \ell \sin^2 \theta_0}{2} (\omega_m / c)^2,$$

where $n(\omega)$ is the refractive index at the frequency ω, and we have arbitrarily introduced the maximum frequency ω_m of the transparency region (the meaning of this frequency is obvious if n = const at $\omega \leq \omega_n$). The intensity of the transition radiation on one interface increases with W, but its absolute value is relatively small (furthermore, the photons emitted are in the main hard and their number is therefore small). The same applies also (see below) to radiation in a vacuum ondulator, i.e., for particle oscillations about a straight-line trajectory under the action of an alternating electric field $\vec{E}$ or of a spatially periodic magnetic field $\vec{H}$ (see [1, 2]; the possibility of using a vacuum ondulator to measure W was indicated in [2, 3]). The purpose of the present article is to point out that when a transparent medium is present in the ondulator [1] the radiation intensity increases sharply under certain conditions. Therefore "an ondulator counter in a medium," as we shall arbitrarily call this system, may be worthy of attention, all the more since such a counter is organically combined with a Cerenkov counter.

The amplitude of the dipole moment of a particle with charge e, produced by a transverse field $E = E_0 \cos\omega_0 t$, is $p_0 = -(e^2E/M\omega_0^2)(Mc^2/W)$. The energy radiated by such a dipole on a path ℓ into a solid angle $d\Omega = \sin\theta d\theta d\phi$ (θ is the angle between the wave vector $\vec{k}$ and the translational velocity $\vec{v}$, and ϕ is the angle between $\vec{E}$ and the projection of $\vec{k}$ on a plane perpendicular to $\vec{v}$) is [1][1)]

$$dS_g(\theta, \phi) = \frac{\omega_0^4 p_0^2 n\ell \{(1-\beta n\cos\theta)^2 - (1-\beta^2 n^2)\sin^2\theta\cos^2\phi\} d\Omega}{8\pi c^4 \beta |1-\beta n\cos\theta|^5}. \tag{1}$$

The radiated frequency is

$$\omega(\theta) = \frac{\omega_0}{|1-\beta n(\omega)\cos\theta|}. \tag{2}$$

In vacuum (at $n = 1$) in the ultrarelativistic case the energy is radiated mainly in an angle $\theta \sim Mc^2/W$, the frequency is $\omega(0) = 2\omega_0(W/Mc^2)^2$, and the total energy is

$$S_{g,b} = \frac{\omega_0^4 p_0^2 \ell}{3c^4}\left(\frac{W}{Mc^2}\right)^4 = \frac{1}{3}\left(\frac{e^2}{Mc^2}\right)\left(\frac{W}{Mc^2}\right)^2 E_0^2 \ell. \tag{3}$$

In a transparent medium at $\beta n > 1$ the radiation energy (1) is concentrated near the Cerenkov angle θ_0. As an estimate we assume that in the region of importance we have $n(\omega)$ = const up to a certain frequency ω_m, and when $\omega > \omega_m$ there is no more radiation (or the radiation does not reach the photomultiplier), owing to absorption. Then as $\beta \to 1$ we obtain from (1) and (2) (we take into account only the principal, second term of (1), which is proportional to $(n^2 - 1)$)

$$S_g = \frac{\omega_0^4 p_0^2 \ell (n^2-1)\sin^2\theta_0}{16c^4}\left(\frac{\omega_m}{\omega_0}\right)^4 = \frac{(n^2-1)^2}{16n^2}\left(\frac{e^2}{Mc^2}\right)\left(\frac{Mc^2}{W}\right)\left(\frac{\omega_m}{\omega_0}\right)^4 E_0^2 \ell. \tag{4}$$

Obviously

$$S_g/S_{g,b} = \frac{3(n^2-1)}{16n^2}\left(\frac{\omega_m}{\omega_0}\right)^4\left(\frac{Mc^2}{W}\right)^4. \tag{5}$$

[1)] For a derivation of an analogous formula see [4]; the result can be easily obtained from formula (73.11) of [5] by changing over from vacuum to a medium with refractive index $n(\omega)$. A somewhat different (spectral) representation of a formula of the type (1), which is suitable for an anisotropic medium, can be found in [6].

In the apparently most favorable case $(n^2 - 1) \sim 1$, the frequency is $\omega_0 \sim 10^{10}$ ($\lambda_0 = 2\pi c/\omega_0 \sim 10$ cm) and $\omega_m \sim 10^{16}$ ($\lambda_m = 2\pi c/\omega_m \sim 2000$ Å). Then the power of the ondulator radiation in the medium exceeds that in vacuum up to values $W/Mc^2 \sim 10^6$; if we disregard the electrons, then this energy region $W/Mc^2 < 10^6$ is at present of greatest interest. In addition, the radiation in the medium propagates mainly at an angle $\theta_0 \sim 1$ and is optical. In a vacuum ondulator, on the other hand, we are dealing with angles $\theta \sim Mc^2/W \ll 1$ and with rather hard radiation and hence with a relatively small number of photons.

Comparing the value of S_g with the energy S_0 of the Cerenkov radiation, we have

$$S_g / S_0 = \frac{(eE_0 c/\omega_m)^2 (n^2 - 1)(\omega_m/\omega_0)^4}{8W^2}. \tag{6}$$

In the case of a magnetic ondulator, which is probably more practical, it is necessary to replace E_0 by the amplitude H_0 of the magnetic field intensity. Obviously, $eE_0 c/\omega_m = eE_0(\lambda_m/2\pi)$ is the work performed by the field E_0 on a path $\lambda_m/2\pi = c/\omega_m$. In our example, $\lambda_m \sim 2 \times 10^{-5}$ cm and $\omega_m/\omega_0 \sim 10^6$. Therefore even for a magnetic ondulator with $H_0 \sim 3 \times 10^4$ Oe we have $S_g/S_0 \sim 10^{21}/W^2$ (eV), i.e., the ondulator radiation is weaker than the Cerenkov radiation when $W > W_c \sim 3 \times 10^{10}$ eV.[2)] Nonetheless, the number of photons emitted in an ondulator in a medium can be sufficient for registration at $W \gg W_c$, say up to an energy $W \sim 3 \times 10^{12}$. It is also important here that the ondulator radiation differs from Cerenkov radiation in its polarization (the electric vector of radiation is directed along $\vec{E}$ or along $\vec{v} \times \vec{H}$, whereas in the case of Cerenkov radiation it lies in the plane of $\vec{k}$ and $\vec{v}$). It is therefore obvious that the same medium can (and should) serve simultaneously as the radiator for the ondulator and for the Cerenkov counters, and the radiation detectors (photomultipliers) should be placed on different generators of the Cerenkov cone. The ondulator radiation can also be distinguished in principle from the Cerenkov radiation by varying (modulating) the parameters E_0 and ω_0.

The fall-off of the radiated energy S_g with increasing particle energy W is undoubtedly the main shortcoming of the ondulator counter in a medium (to be sure, when account is taken of secondary particles, including recoil electrons, this fall-off should slow down; allowance for the secondary particles is the main task of further research). It seems to us, nevertheless, that the combination of a Cerenkov counter with an ondulator counter can turn out to be an effective method.

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[2)] Formula (6) is rigorously valid only if $S_g/S_0 \ll 1$, since we have used above the dipole approximation $|x_0| = |p_0/e| \ll c/\omega_m$ (this circumstance was pointed out to the author by V.N. Tsitovich).