SOLUTION OF THE NUCLEAR THREE-BODY PROBLEM WITH RELATIVISTIC POTENTIALS

V.F. Demin and V.D. Efros
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At the present time there are several so-called realistic potentials that describe accurately the totality of the experimental data on a two-nucleon system. It is of interest to attempt to describe the properties of systems containing several nucleons by starting from such potentials. A comparison of the calculated properties with the experimental ones allows us to assess the validity of pair NN potentials as models of nuclear forces, and also the role of different parts of an NN potential.

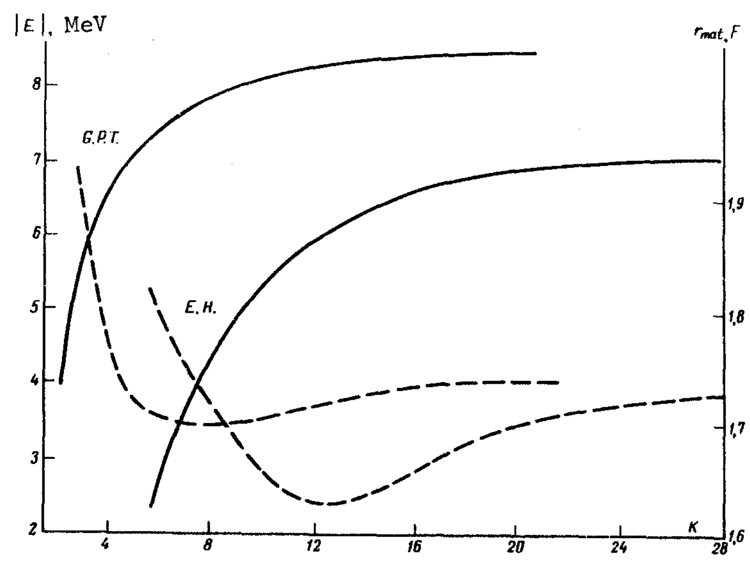
We report here the resultant properties of the ^3H nucleus with the following two "realistic" NN potentials, fitted to the NN-scattering phase shifts in a large energy interval, namely, the potential of the Eikemeier and Hackenbroich [1] and the potential of Gogry, Pires, and De Toureil [2], which will henceforth be denoted E.H. and G.P.T. All the parts of these potentials, without exceptions, were taken into account in the calculations.

Our method is based on the hyperspherical formalism. We denote by ρ and Ω_6 the radius and the angle variables on a hypersphere in six-dimensional space, and by $\{\sigma^{(i)}, \tau^{(i)}\}$ the spin-isospin variables of the particles. We seek the Ψ function of the problem in the form of the expansion

$$\Psi = \rho^{-5/2} \sum_{K, \alpha, n} c_{K\alpha n} R_n(\rho) \Gamma_{K\alpha}(\Omega_6, \{\sigma^{(i)}, \tau^{(i)}\}), \tag{1}$$

where $\Gamma_{K\alpha}$ is the hyperspherical basis. For the time being, we take into account in the basis $\Gamma_{k\alpha}$ only the so-called "potential harmonics" (PH) [3 - 5]. The general method of constructing the PH, which makes it possible, in particular, to consider an orbital momentum $L \neq 0$ and non-central NN forces, is given in [5]. For the system $R_n(\rho)$ we choose the functions

$$R_n(\rho; b, s) = N(b\rho)^{s/2} L_n^s(b\rho) \exp(-b\rho/2), \tag{2}$$



where $L_n^s(x)$ are associated Laguerre polynomials, and b and s are parameters chosen to ensure the fastest convergence with respect to n.[1)]

The figure shows the binding energy $E(^3H)$ calculated by us and of the material rms radius r_{mat} as functions of the number of hyperspherical functions taken into account in the expansion (1). The points with abscissas K_f correspond to allowance for PH with K = 0, 2, ..., K_f. The largest value of K taken into account in the expansion (1) was K_{max} = 28 for the E.H. potential and K_{max} = 20 for the G.P.T. potential. It is seen from the figure that a practical convergence was attained for $E(^3H)$ and r_{mat} with these values of K_{max}.

The final results of the calculations of $E(^3H)$ and of the partial weights of the spatially-symmetrical component of the Ψ function with L = 0 (S), the mixed-symmetry components with L = 0 (S'), and the components with L = 2 (D) are listed in Table 1.

TABLE 1

Potential	E, MeV	r_{mat}, F	$P(^3H)$, %			P_D deut., %
			S	S'	D	
E.H.	7.06	1.73	91.3	0.7	8.0	6.2
G.P.T.	8,50	1,74	94.6	0,6	4,7	3.8

We note that the calculations have established that the S' component of the Ψ function is the result of a difference of two contributions, one due to the coupling with the D state (via the potential $V_t^{(1)}$, see the footnote of Table 2), and one due to coupling with the S state (via the potential V_{31} - V_{13}). As a result, the partial weight of the S' component should be quite sensitive to details of the NN potential.

To explain the dependence of the results obtained for $E(^3H)$ on the structure of the NN potential, we have calculated the mean values of the kinetic energy and of the individual parts of the NN potential using the obtained Ψ function of ^3H. The results are shown in Table 2. The following circumstances are remarkable: (a) The extremely small contribution of the odd potentials to $E(^3H)$. (b) $E(^3H)$ is the result of subtraction of two large quantities, the kinetic and potential energies. This makes $E(^3H)$ sensitive to details of the NN potential. (c) The contributions of the kinetic energy and of individual parts of the NN potential to $E(^3H)$ for different realistic local potentials differ strongly from one another. Thus, these potentials are not "close" in the case of ^3H. It is of interest that, nevertheless, all the realistic local NN potentials considered to date[2)] give values of $E(^3H)$ that are close to one another and to the experimental value (the difference in the values of $E(^3H)$ is of the order of 1 - 2 MeV and is apparently immaterial in view of the indicated sensitivity to the details of the potential).

[1)] It can be assumed that a system of functions in the form of (2) with s that depends on K, say s = α + βK, may even be more suitable.

[2)] We have in mind, in addition to the results given above, also the known results for the potentials of Hamada and Johnston, Bressel, Kerman, and Rouben, and Reed.

Table 2

$\langle \Psi \| \ldots \| \Psi \rangle$	E, MeV	
	E.H.	G.P.T
Kinetic energy	41,7	29,4
$V_{31} + V_{13}$	- 22,3	- 27,9
$V_t^{(1)}$	-23,6	- 9,5
$V_{LL}^{(1)}$	–	- 0,3
$V_{LS}^{(1)}$	- 2.7	-0,2
$V_{33} + V_{11}$	0.1	0,02
$\|V_{LL}^{(3)}\|$, $\|V_t^{(3)}\|$, $\|V_{LS}^{(3)}\|$	$\leqslant$ 0.1	$\leqslant$ 0,1
Sum	- 7.06	- 8,50

Note. $V_{ij} = V_{2S+1,\ 2T+1}^{center}$; $V_t^{(1)}$ ($V_t^{(3)}$) is the tensor singlet (triplet) potential. The notation is analogous for the LS and $(LS)^2$ potentials ($V_{LS}^{(i)}$ and $V_{LL}^{(i)}$).

We have also assessed the relative role of the NN interactions in states with different orbital angular momenta ℓ_{ij} of the particle pairs. To this end, additional calculations were performed, in which the NN interaction for all but the tensor forces was taken into account only in states with $\ell_{ij} \leq \ell_{max}$, and for the tensor forces we took into account in addition also the transitions $\ell_{max} \rightleftarrows \ell_{max} + 2$. Table 3 lists the results for $E(^3H)$ as a function of ℓ_{max} for the E.H. potential. We can conclude that the approximation $\ell_{max} = 2$ is already sufficiently accurate.

Table 3

ℓ_{max}	E, MeV
0	- 6,401
2	- 7,059
4	- 7,063
6	- 7.063

We note in conclusion that we have shown in the present paper that the solution of the nuclear three-body problem with "realistic" NN forces is possible in practice within the framework of the hyperspherical approach.

We propose to report in future papers the results of calculations of the properties of ^3H with Bressel-Kerman-Rouben and Reed potentials and the results of calculations of ^4He with realistic NN forces.

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[2] D. Gogry, P. Pires, and R.De Toureil, Phys. Lett. 32B, 591 (1970).
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ERRATA

Article by A. Yu. Aleksandrov et al., Vol. 16, No. 4:

On p. 147, lines 17 - 18 from top, read ...Cs_2SbCl_6(B) and $RbSbCl_6 \cdot 2Rb_3SbCl_6$(A).... instead of ... Cs_2SbCl_6(A) and $RbSbCl_6 \cdot 2Rb_3SbCl_6$(B).

On p. 148, lines 16 - 17 from the bottom, read ...$kT_n \sim Vz$... instead of ...$kT_n \sim V_z$...

Article by L. E. Gendenshtein and A. B. Kaidalov, Vol. 16, No. 4:

On p. 177, line 22 from top, read ... $C_R \approx 1/30 C_1 \approx 5C_p$ ... instead of $C_R \approx 1/30$, $C_1 \approx 5C_p$.