OBSERVATION OF EXPLOSIVE INSTABILITY OF PARAMETRICALLY COUPLED WAVES

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Explosive instability is one of the most interesting effects occurring when parametrically coupled waves interact in nonlinear (quadratic) non-equilibrium media [1 - 4]. This instability is manifest by a simultaneous increase of the amplitudes of all the interacting waves and leads, under the customarily employed idealization, to their becoming infinite within a finite time or over a finite distance (divergence of the type $1/(t_\infty - t)$). The most frequently cited example of a non-equilibrium medium in which explosive instability is possible is a plasma through which an electron beam passes [2 - 4]. In the transparency region, longitudinal waves having both positive and negative energy can exist in such a medium [1]. Inasmuch as a wave with negative energy increases as it gives up energy to the ordinary wave, the nonlinear process accompanied by the decay of such a wave leads to explosive instability. We note that the instability occurring in the case of linear coupling of waves carrying "kinetic power" [5] and having opposite signs has long been in use in electronics; in particular, the traveling wave tube operates on this principle [6].

In spite of the large number of papers devoted to explosive instability, we know of no experiments in which it was observed.

We present here the results of an experimental investigation of explosive instability of electromagnetic waves.

It is easy to verify that the effect of explosive instability for electromagnetic waves having positive energy is possible in a medium whose imaginary component of the dielectric constant is proportional to the field. The equations for the amplitudes of parametrically-coupled waves, satisfying the synchronism condition

$$\omega_1 + \omega_2 = \omega_3, \qquad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3, \tag{1}$$

assume in such a medium the form

$$\frac{\partial a_{1,2}}{\partial t} + v\nabla a_{1,2} = \sigma_{1,2} a_3 a^*_{2,1}, \qquad \frac{\partial a_3}{\partial t} + v\nabla a_3 = \sigma_3 a_1 a_2. \tag{2}$$

Unlike the ordinary equations of parametrically-coupled waves, in this case the right-hand sides of all equations are of the same sign. Instead of the Manley-Rowe equations for the spatially-homogeneous fields, we obtain the integrals

$$\sigma_3|a_1|^2 - \sigma_1|a_3|^2 = C_1; \quad \sigma_3|a_2|^2 - \sigma_2|a_3|^2 = C_2; \quad \sigma_1|a_2|^2 - \sigma_2|a_1|^2 = C_3. \tag{3}$$

It follows from them directly that the amplitudes of all three waves should increase simultaneously in the case of nonlinear interaction. The energy consumed by the waves as their amplitudes increase is drawn from a source that maintains the medium in the non-equilibrium state (in this case - the dependence of the conductivity of the medium on the square of the field).

Naturally, Eqs. (2) no longer hold at large field amplitudes and in the case of rapid variation of these fields. In order for the model to be correct it is necessary to take into account nonlinearities of higher order, particularly the one cubic in the field, or else a finite rate of energy consumption

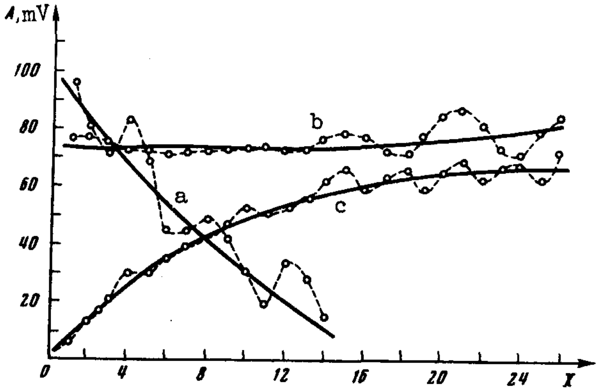

Fig. 1

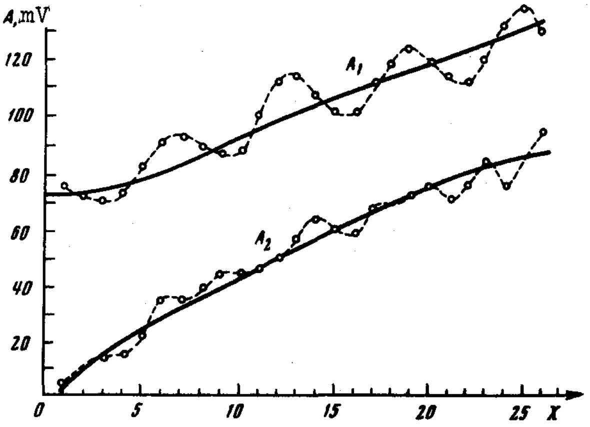

Fig. 2

from the source that maintains the non-equilibrium character (activity). Both factors limit the amplitudes of the interacting waves and stabilize the explosive instability. Thus, it is impossible to observe the explosion directly in a real medium, but the main features of explosive instability can be observed experimentally. The main characteristic of the instability should be taken to be not a singularity of the type $1/(t_\infty - t)$, but a simultaneous growth, as a result of the nonlinear process, of the amplitudes of the interacting waves. Such an instability can obviously be maintained also in the presence of linear dissipation.

Explosive instability in a nonlinear medium was observed in the radio band in a two-wire transmission line with nonlinear leakage, in which the dependence of the current on the voltage was in the form $j = -\sigma_L u^2$. The nonlinear-leakage elements were tunnel diodes whose operating points were at the maxima of the characteristics. We emphasize that the realization of a non-equilibrium medium with the aid of tunnel diodes presupposes the presence of an external energy source - a bias source that determines the position of the diode operating point.

The experiment demonstrating the existence of explosive instability consists of the following. First, one of the monochromatic waves of frequency ω_1, ω_2, or ω_3 was launched in the investigated system, into which linear dissipation was introduced independently of the tunnel diode. It was then observed that the wave attenuates exponentially as it travels along the line. This was followed by application of a pair of waves with frequencies ω_i and ω_j. Since the synchronism conditions (1) were satisfied, a third wave of combination frequency was produced, and the amplitudes of all waves increased simultaneously. The rate of increase depended on the limiting values of the amplitudes. In the case when the synchronism conditions were not satisfied, both waves attenuated independently of each other.

A similar experiment was performed for the degenerate case - interaction of the fundamental wave and the second harmonic. Since the second harmonic is always generated in the presence of a wave ω if the synchronism conditions are satisfied, these conditions were artificially violated in order to determine the independent behavior of these waves. Figure 1 (curve a) shows the damping of the waves ω and 2ω in the line in the absence of interaction. Curves b and c of the same figure show that when the synchronism conditions are restored the amplitudes of these waves increase simultaneously as a result of the nonlinear interaction, thus proving the existence of explosive instability1).

In the absence of linear dissipation (i.e., when the additional resistances are disconnected), the amplitudes of the waves ω_1 and $\omega_2 = 2\omega_1$ were constant along the line if the synchronism conditions were not satisfied. The behavior of the amplitudes of these waves in the case of complete synchronism is shown in Fig. 2. The simultaneous growth of the waves, which demonstrates the explosive character of the nonlinear interaction, is analogous to the growth of the amplitudes of coupled waves with energies of opposite signs in a non-equilibrium plasma.

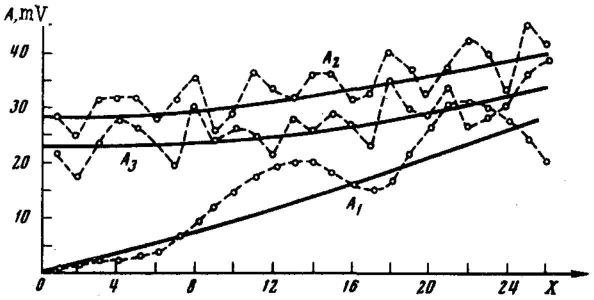

Fig. 3

Figure 3 shows the results of an investigation of the non-degenerate process in a system where the linear dissipation is small. We see that in the presence of two waves there is produced a third wave of combination frequency, generation of which is accompanied by the growth of all three waves (when one wave propagated in this system, its amplitude decreased slowly).

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BREAKDOWN AND RECOMBINATION KINETICS CONNECTED WITH THE EXCITED STATES OF A SHALLOW DONOR IN n-Ga

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We report here the results of an investigation of the kinetics of impurity breakdown and recombination, connected with the first and higher excited states of a shallow donor in n-Ge. Simultaneous registration of the current-voltage characteristics leads to the conclusion that an appreciable contribution is made by impact ionization of the excited states to the total intensity of the breakdown process.

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1) The spatial beats are connected with the presence of weak reflected waves.