DIAGRAM K-MATRIX APPROACH TO pα SCATTERING

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Diagram and dispersion methods have recently found extensive use in the theory of nuclear reactions (cf., e.g., [1, 2]). The common basis of these methods is the assumption that the reaction amplitude M is an analytic function of the kinematic variables (of the scattering angle or energy), and that the main contribution to M is made by the singularities, with respect to these variables, that are closest to the physical region. In the case of binary reactions (A + x → B + y), the assumption that the nearest singularities play the predominant role is certainly valid for the partial amplitudes M with sufficiently large values of ℓ [3], but for small ℓ it may not be satisfied. The use of this assumption for small ℓ can lead, in particular, to serious violations of the unitarity relation. One of the methods of ensuring unitarity is to use the formalism of the reaction K matrix, which is connected with the S matrix by the relation

$$S = \left(1 - \frac{i}{2}K\right)/\left(1 + \frac{i}{2}K\right). \qquad (1)$$

The S matrix is automatically unitary for any Hermitian K matrix.

We consider the single-channel case of pure elastic scattering of nonrelativistic spinless particles: A + x → A + x. Then, assuming

$$M(E,z) = \sum_{\ell=0}^{\infty} (2\ell+1) M_\ell(E) P_\ell(z), \quad K(E,z) = \sum_{\ell=0}^{\infty} (2\ell+1) K_\ell(E) P_\ell(z) \qquad (2)$$

($z = \cos\theta$, E and θ are the kinetic energy and the scattering angle in the c.m.s.), we readily obtain from formula (1):

$$M_\ell = K_\ell / (1 + i\mu p K_\ell / 2\pi), \qquad (3)$$

μ and p are the reduced mass and the relative momentum of the colliding particles[1)].

The normalization of the amplitude M(E, z) is determined by the relation

$$d\sigma/d\Omega = (\mu/2\pi)^2 [M(E,z)]^2 \qquad (4)$$

where dσ/dΩ is the differential scattering cross section. The K matrix was initially defined only in the physical region of the reaction. With the aid of relations (1) and (3), however, it can be continued analytically into the unphysical region. Considering formula (3) as $\ell \to \infty$ and using the results of [3], it is easy to verify that if M(E, z) has a singularity in z at $z = z_0$, then K(E, z) also has a singularity at $z = z_0$. In particular, if M(E, z) has a pole in z, then K(E, z) also has a pole in z at the same point and with the same residue. Therefore, following the spirit of the dispersion approach and wishing at the same time to ensure unitarity,

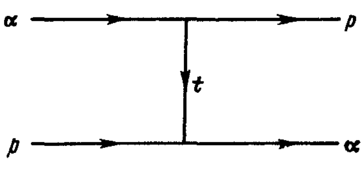

Fig. 1. Pole diagram for pα scattering.

1) Here and elsewhere $\hbar = c = 1$.

it is natural to assume that the K matrix is determined by the contribution made by the z-plane singularities closest to the physical region, which are singularities of the amplitudes of the simplest Feynman diagrams.

Let us apply these considerations to large-angle elastic pα scattering at energies below the α-particle disintegration threshold. In this case the singularity closest to the boundary of the physical region (z = 1) is the pole corresponding to the triton-transfer mechanism (Fig. 1); the remaining singularities are much farther away. Choosing as K(E, z) the amplitude of the diagram of Fig. 1, we obtain[2)]

$$K(E, z) = -m_t G^2/[2p^2(z + \zeta)], \quad K_\ell(E) = (-1)^{\ell+1} m_t G^2 Q_\ell(\zeta)/2p^2, \tag{5}$$

where $\zeta = (1/2)(m_\alpha/m_p + m_p/m_\alpha) + m_t\varepsilon/p^2$, $\varepsilon = m_t + m_p - m_\alpha$, m_i is the mass of particle i, $Q_\ell(\zeta)$ are Legendre functions of the second kind, and $G \equiv G_{\alpha tp}$ is the vertex constant [4] for the vertex $\alpha \to t + p$.

The differential elastic pα scattering cross section was calculated for different energies and at different values of the constant G. By way of an example, Fig. 2 shows the theoretical curves and the experimental points [5] for the differential cross section for elastic pα scattering at an incident proton energy 20.62 MeV. Curve 1 of Fig. 2 was obtained with the aid of formulas (2) - (5) at G^2 = 7 F. We see that although this curve does duplicate qualitatively the course of the experimental cross section, there is no quantitative agreement. The reasons for this discrepancy lie, first, in the fact that at small ℓ, especially at ℓ = 0, singularities more remote than the pole, corresponding to other mechanisms, can make a noticeable contribution to the partial amplitudes. This contribution can be approximately taken into account by adding to the partial S amplitude M_0 a complex constant and by adjusting the latter by the χ^2 method. Curve 3 of Fig. 2, which was calculated by this method, describes well the experimental data at $\theta \gtrsim 100°$. We note that the results of the calculations are sensitive to the value of G^2, and the best agreement with experiment is obtained at G^2 = 7 F. This is illustrated by curve 2, which was calculated by the same method as curve 3, but with G^2 = 4 F; it deviates noticeably from the experimental points. A fair agreement between theory and experiment at the same value G^2 = 7 F is obtained also at other proton energies.

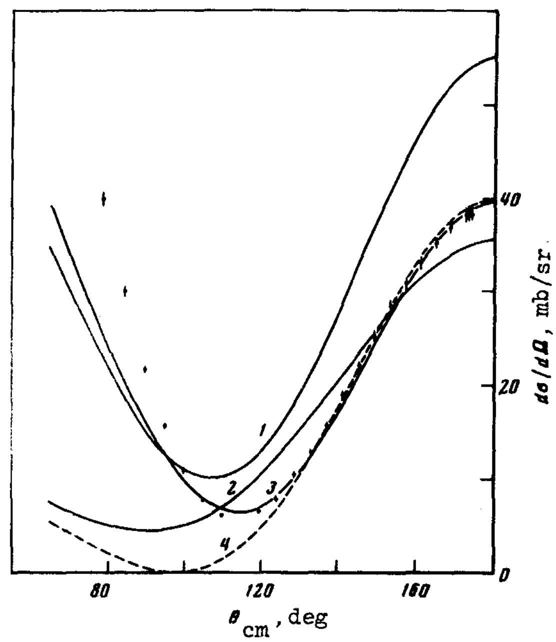

Fig. 2. Comparison of theoretical differential cross sections with experiment for pα scattering at a proton energy 20.62 MeV.

For comparison, Fig. 2 shows also the results of calculations of the differential pα scattering cross section at 20.62 MeV within the framework of the peripheral model with the pole mechanism [2, 6] (dashed curve 4). We see that the proposed approach, within the

[2)]The presence of proton spin leads in this case only to the appearance of the trivial factor $\delta_{\mu_i \mu_f}$, which we shall omit (μ_i (μ_f) is the projection of the proton spin in the initial (final) state).

framework of the same pole mechanism, improves the agreement between theory and experiment in comparison with the peripheral model. It is interesting to note that in the case of a pheripheral model the best description of the experiment is also obtained at $G^2 = 7$ F. This value of G^2 agrees with the results of an analysis of the reaction $He^3(d, p)He^4$ within the framework of the peripheral model [2] ($G^2 = 7.1$ F); it is somewhat lower than the value $G^2 = 11.3$ F obtained in [7] for the constant of the isotropically-similar process $\alpha \to \tau + n$ by using the dispersion relations for the amplitude of the elastic forward $n\alpha$ scattering. The vertex constant G_{ABC} for the decay (fusion) via the channel $A \to B + C$ is connected with the coefficient in the asymptotic wave function of this channel, and contains important spectroscopic information; unlike the customarily employed reduced width θ, its definition does not contain a model quantity such as the channel radius.

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PHOTOPRODUCTION OF CHARGE PIONS ON NUCLEI AND VIOLATION OF THE VECTOR DOMINANCE MODEL

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The cross sections for the photoproduction of charged pions on nuclei, calculated in [1] within the framework of the vector dominance model (VDM), using the approximate procedure for taking into account the effective nucleon numbers N^{eff} on the energy [2] are on the whole in satisfactory agreement with the experimental data [3]. However, when a more accurate account is taken of the energy dependence of N^{eff} (see Eq. (1) below) within the framework of the VDM, a noticeable discrepancy is observed between the predictions of the theory and experiment (see curve 1), thus indicating either violation of the VDM, or incorrectness of the scheme used to describe the interaction of the particles with the nuclei. An indication that the VDM is apparently violated should be the discrepancy between the values of the coupling constant $\gamma_\rho^2/4\pi$ obtained from an analysis of the photoproduction of $\pi^\pm$ mesons on nucleons [4] ($\gamma_\rho^2/4\pi \simeq 0.3$), and from the data on the photoproduction of ρ^0 mesons on nucleons [5] ($\gamma_\rho^2/4\pi \simeq 0.7$) observed in recent years. Consequently, the comparison of the theoretical calculations with the experimental data on the photoproduction of charged pions on nuclei can serve only as a verification of the theory of incoherent production of particles on nuclei. Assuming, just as in the VDM, that the amplitudes of the processes of photoproduction of particles on nucleons are proportional to

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