molecules that resonate with the coherent light wave (the factor q usually lies in the interval 10^{-1} for simple molecules to 10^{-3} for complex ones), the relative amplitudes of the narrow resonances lie in the interval 0.1 - 10%. A very important factor is the automatic tuning of the resonance frequencies when the light-wave frequency is scanned along the Doppler contour of the optical transition, the absolute tuning range of the γ-radiation resonance frequencies being ω_γ/ω_0 larger than the range of the optical transitions.

Since the coefficient of the resonant absorption on nuclear transitions is small for a rarefied gas, the proposed method is most convenient for the production of narrow γ-radiation lines with tunable frequency. The intensity of the emission line with width Γ_{nuc} is narrower by a factor 10^2 than the Doppler width from a tube with gas filled with molecules with radioactive nuclei at a pressure 10^{-2} Torr ($q = 10^{-2}$, $G \simeq 1$, tube length 100 cm, diameter 1 cm, observation solid angle 10^{-4} sr) is equal to 10^7 τ gamma quanta/sec, where τ is the radiative lifetime of the excited state of the nucleus.

The proposed method permits the performance of a number of fundamental experiments. For example, by tuning a narrow γ-radiation resonance $\omega^{(1)}_{nuc} = \omega_\gamma - \Delta_\gamma - \vec{k}_0\cdot\vec{v}(\omega_\gamma/\omega_0) + \omega_0$ in the vicinity of the $\omega_\gamma + \omega_\gamma$ absorption line of the nucleus and the target, it is possible to measure exactly the recoil energy of the nucleus and the shape of the phonon spectrum [3]; the latter permits a detailed investigation of the vibrations of the nuclei in the crystal. Since the quantity $\vec{k}_0\cdot\vec{v}$ can be measured with very high accuracy by measuring the deviation Ω of the light-wave frequency from the center of the Doppler contour, the frequency of the γ-radiation resonance can be tuned with accuracy $\delta\omega_\gamma/\omega_\delta = \delta\Omega/\omega_0$, which can reach in principle 10^{-11} - 10^{-13}. This makes possible absolute measurements of the γ-quantum energy with accuracy ω_γ/ω_0 (10^{-11} - 10^{-13}), which can reach, in principle, values 10^{-8}, and thus relate, with the same accuracy, the energy scales of the optical and γ-ray bands. Naturally, the proposed method makes possible γ spectroscopy in the region of nuclear transitions $10^{-9} < |(\omega_{nuc}/\omega_\gamma) - 1| < 10^{-5} - 10^{-6}$, which has heretofore been inaccessible to modern methods. We note that it is applicable also for transitions in which the Mossbauer effect does not occur (for example, at a γ-quantum energy much higher than 100 keV).

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CONTRIBUTION OF NUCLEON-TRITIUM CHANNEL TO THE 0^+ STATE OF FOUR PARTICLES

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We discuss in this article the cluster approximatinn (CA) "4 = 3 + 1" in the integral equations for four particles. We have considered the variant of integral equations proposed in [1], which coincide for four particles, accurate to the free terms, with equations of the Omnes type [2]. For a discrete spectrum, these equations, written out for the components of the form factor $F = G_0\Psi$, take the form

$$\lambda F^{\alpha a} = \tilde{T}^{\alpha}_{a} G_0 \sum_{\beta\neq\alpha,\ \beta\supset b} F^{\beta b}, \qquad (1)$$

where a, b = (ijk) (ℓ), ij(kℓ) are different subdivisions of the four particles into two groups, $\alpha \in a$ and $\beta \in b$ are particle pairs entering in one subdivision group a or b. In formula (1), $\lambda = \lambda(z)$ (z is an energy parameter) is an eigenvalue, V_0 is the free Green's function, and $\tilde{T}^{\alpha}_a = \sum_{\beta \in \alpha} W^{\alpha\beta}_a$ is the sum of the bound amplitudes for the subsystems 3 + 1 and 2 + 2. The CA "4 = 3 + 1" will be defined as the approximate equations obtained by setting the amplitudes corresponding to the subdivision "4 = 2 + 2" equal to zero: $F^{\alpha a} = \tilde{T}^{\alpha}_a = 0$ for a = (ij)(kℓ). In this approximation, we neglect the nuclear singularities connected with the threshold for the disintegration into two deuterons; therefore the asymptotic wave function does not contain terms corresponding to the d - d channel[1]. Introducing separate Jacobi coordinates for each function $F^{a\alpha}$ [a = (ijk)(ℓ)] (see [1], formula 33)), we obtain for $F(z) \equiv F(\vec{k}, \vec{p}, \vec{q}\,|z)$ the equation

$$\lambda F = \left(\frac{3}{2\sqrt{2}}\right)^3 \int [\, W(1) + W(2) + W(3)]\, \Delta^{-1} [\, F(2) + F(3)]\, d^3k'\, d^3q', \qquad (2)$$

where $W = W^{(1)} + 2W^{(2)}$ (the indices 1 and 2 pertain respectively to the diagonal ($\alpha = \beta$) and nondiagonal ($\alpha \neq \beta$) amplitudes),

$$W(n) = W\left(\mathbf{k}, \mathbf{p};\ \Lambda_n[\mathbf{k}', \mathbf{Q}_1] \,|\, z - \frac{q^2}{2m}\right) \quad (n = 1, 2, 3),$$

$$F(n) = F(\Lambda_n[\mathbf{k}', \mathbf{Q}_2],\ \mathbf{q}' \,|\, z) \quad (n = 2, 3), \qquad (3)$$

$$\Delta = z - \frac{k'^2}{2m} - \frac{Q_1^2}{2m} - \frac{q^2}{2m} = z - \frac{k'^2}{2m} - \frac{Q_2^2}{2m} - \frac{q'^2}{2m};$$

$$\mathbf{Q}_1 = \frac{\mathbf{q} + 3\mathbf{q}'}{2\sqrt{2}}, \quad \mathbf{Q}_2 = \frac{\mathbf{q}' + 3\mathbf{q}}{2\sqrt{2}}.$$

The matrices Λ_n in (3) act on the column $(\vec{\mathcal{P}}/k)$ ($\vec{\mathcal{P}} = \vec{Q}_1, \vec{Q}_2$); in a basis (123) the matrices Λ_1, Λ_2, and Λ_3 correspond to the permutations P_{12}, P_{13}, and P_{23}, respectively. For identical particles, W and F in (2) are even under the substitution $\vec{k} \to -\vec{k}$. The matrix Λ_1 in (2) can therefore be replaced by a unit matrix.

It is easy to generalize Eq. (2) to take into account the spin isospin (ST) variables. Let us consider the equations for the state with S = T = 0. Confining ourselves to even potentials, we can expand $F^{\alpha a}$ of (1) in terms of two ST functions characterized by the ST values s_α and i_α of the singled-out pair α, and also σ_α and τ_α of the singled-out triad of particles. The coefficients in this expansion will be designated F_i, where i = 0 and 1 are the values of the isospin i_α, and the value of s_α is determined uniquely from the condition $i_\alpha + s_\alpha = 1$. Analogously, the amplitudes $W^{\alpha\beta}_a$ of (1) can be represented in the form of projection operators with eigenvalues $W^{(1,2)}_{ij}$, where i and j denote the isospins of pairs α and β. As a result we obtain

[1] The class of diagrams summed within the framework of the given CA depends also on the equation in which this approximation is carried out. Thus, for example, in the CA "4 = 3 + 1," the classes of the diagrams summed in (1) and in the Yakubovskii equations [3] are different.

$$\lambda F_i = \left(\frac{3}{2\sqrt{2}}\right)^3 \int d^3k' d^3q' \left\{ \left[\frac{1}{4} W^s_{io} + \frac{3}{4} W^s_{i1}\right] [F_o(2) + F_o(3)] + \right.$$

$$+ \left[\frac{3}{4} W^s_{io} + \frac{1}{4} W^s_{i1}\right] [F_1(2) + F_1(3)] + \frac{3}{4} [W_{io}(2) - W_{i1}(2)][F_o(3) -$$

$$\left. - F_1(3)] + \frac{3}{4} [W_{io}(3) - W_{i1}(3)][F_o(2) - F_1(2)] \right\} \Delta^{-1}, \tag{4}$$

where

$$W^s_{ij} = W_{ij}(1) + W_{ij}(2) + W_{ij}(3), \quad W_{ij} = W^{(1)}_{ij} + 2W^{(2)}_{ij}.$$

Equations (4) are a system of multidimensional integral equations. The use of a separable expansion for the amplitudes yields a system of one-dimensional integral equations. We shall consider a method, based on an expansion of the Hilbert-Schmidt (HS) type [4 - 6], for making the amplitudes W_{ij} separable. The first term in this expansion contains the contribution of a pole connected with the 4 = 3 + 1 threshold, and also part of the contribution of two- and three-particle cuts. Allowance for the remaining terms corresponds to supplementing the approximate expression for W_{ij} until it becomes unitary; as will be shown below, it results in a small correction in the problem of the discrete spectrum of four particles.

The HS expansion for the three-particle amplitudes was considered in [7] (see also [8]). For a separable potential $v_i(k, k') = -(\lambda_i/2m)g_i(k)g_j(k')$, the expansion for the amplitudes W_{ij} takes the form

$$W_{ij}(k, p; k', p' | z) =$$

$$= -\frac{1}{4\pi} \sum_m \frac{\eta_m(z)}{1 - \eta_m(z)} w_{im}(k, p | z) w_{jm}(k', p' | z), \tag{5}$$

where

$$w_{im}(k, p | z) = \sqrt{\frac{\lambda_i}{2m}}\, g_i(k)\, w_{im}(p | z)\, d_i^{-1}\left(z - \frac{p^2}{2m}\right),$$

$\eta_m(z)$ are the eigenvalues and $w_{im}(p|z)$ are eigenfunctions given by the formulas of [7] (formulas (16) and (19)). Substituting in (4) the expansion[2)]

$$F_i(k, p, q | z) = \sum_m w_{im}\left(k, p | z - \frac{q^2}{2m}\right) c_m(q | z) \tag{6}$$

we obtain a system of one-dimensional integral equations for the function c_m. The kernel of these equations is determined by the quadruple integral $\int dk \int d^3k$. Such a representation of the nucleus is quite inconvenient when solving integral equations, and we therefore use in the calculations the expansion of the eigenfunctions in series of K-harmonics [9] (see [7], Sec. 4). Using this expansion, we can carry out analytically the integration with respect to Ω_k and represent the kernel in the form of a sum of double integrals. The detailed

[2)]We have taken into account the contribution made to the state 0^+ only by the amplitudes W_{ij} with L = 0. Therefore the function F_i in (6) depends only on the moduli of the vectors $\vec{k}$, $\vec{p}$, and $\vec{q}$.

procedure is described in a preprint of our Institute.

We have used in the calculations the Yamaguchi separable model, which corresponds to the low-energy NN-scattering parameters $a_t = 5.372$ F, $a_s = -22.827$ F, $r_{ot} = 1.715$ F, and $r_{os} = 2.704$ F. The eigenvalues $\eta_m(z)$ break up into two classes, one containing $\eta_m^{(+)}(z) > 0$ and the other $\eta_m^{(-)}(z) < 0$. The three-particle system has one bound state: $\eta_1^{(1)}(z_0) = 1$, $z_0 = -11.03$ MeV. The results of the calculations[3)] for the 0^+ states of four particles points to a rather rapid convergence of the HS expansion as a numerical method in the four-body problem. Thus, the first HS term makes it possible to calculate the eigenvalue $\lambda_1(z)$ with accuracy $\sim 0.1 - 1.5\%$ in the interval $11.03 \leq -z \leq 45$ MeV (the error increases with increasing $-z$). The value of the energy of the ground state z_α is obtained in this case with accuracy $\sim 1.5\%$ (see the table). The accuracy of the calculation of $\lambda_2(z)$, and consequently of the possible excited states is somewhat worse and varies from 5% at the threshold to 50% at $z = -45$ MeV.

Binding energy of the ground state

Number of equations	No. of included $\eta_m^{(+)}$	No. of included $\eta_m^{(-)}$	z_α	PA	z_α
1	1	0	39,061	$K_{max} = 0$	39,700
3	2	2	39,645	$K_{max} = 2$	37,250
5	3	2	39,647	–	–

The excited 0^+ state appeared in our calculations as a bound state lying almost exactly at the threshold of the "4 = 3 + 1" disintegration. It is possible that when the K-harmonics with $K_{max} > 2$ are taken into account, this "4 = 3 + 1" state will vanish, but $\lambda_2(z_0)$ will remain quite close to unity. The table lists also the results of calculations in the pole approximation (PA), which works well in the calculation of $\lambda_1(z)$ (with an error $\sim 1.3\%$ at $z = z_0$ and $\sim 10\%$ at $z = -45$ MeV, the error in z_α being $\sim 7\%$), and much worse for $\lambda_2(z)$ (the error here is $\sim 50\%$). The excited level does not appear in the PA.

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ANGULAR DEPENDENCE OF THE ASYMMETRY OF THE CROSS SECTION FOR THE REACTION $\gamma p \to \Delta^{++}\pi^-$ AT A PHOTON ENERGY 650 MeV

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The behavior of the asymmetry of the cross section $\Sigma = (\sigma_\perp - \sigma_\parallel)/(\sigma_\perp + \sigma_\parallel)$ of the photoproduction of the Δ^{++} isobar

$$\gamma + p \to \Delta^{++} + \pi^- \tag{1}$$

is a very sensitive criterion for different theoretical descriptions, and permits a choice between model representation of the mechanism of the reaction in the case when all the models give a good description of the total and differential cross sections. To this end it is necessaty to study the behavior of the asymmetry in a wide range of angles and energies of the photons. No such study of the angular and energy behavior of the asymmetry near the threshold have been made, and there are no experimental data.

We present here the results of a measurement of the asymmetry of the cross section of the reaction (1) on a linearly-polarized beam of photons at an energy 650 MeV in the π^--meson emission angle interval 45 - 120° in the c.m.s. The polarized photons were obtained from an individual reciprocal-lattice point of a diamond single crystal [1, 2].

The measurements were performed simultaneously with two magnetic spectrometers [3] with solid angles 1.3×10^{-3} and 8.2×10^{-3} sr. The π^- mesons were detected with scintillation-counter telescopes, and the momentum range was 9.4%.

Figure 1 shows the dependence of the pion yield on the orientation of the diamond crystal. One spectrometer was used to measure the asymmetry of the π^- mesons from the reaction of binary production at an angle $\theta = 90°$ and a photon energy $E_\gamma = 650$ MeV, while the other spectrometer measured simultaneously the asymmetry of the cross section of single photoproduction of π^+ mesons for the same energy and angle. The asymmetry for single production under these conditions, according to our measurements, is $\Sigma = 0.6 \pm 0.05$. In the binary photoproduction reaction, a near-zero symmetry is observed in this case.

The asymmetry is defined by the relation

$$\Sigma = \frac{1}{P}\,\frac{R-1}{R+1},$$

where R = C /C is a quantity obtained directly from the measurements, equal to the ratio of the π^--meson yields from the reaction $\gamma + p \to p + \pi^+ + \pi^-$ when the polarization vector of the photon beam is perpendicular and parallel to the reaction plane, and P is the effective polarization of the photon beam. It can be obtained by using the fact that the value of the polarization of the photon