TEMPERATURE DOMAINS IN SUPERCONDUCTOR FILMS

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It is shown that temperature domains can be produced in thin superconductor films as a result of the released Joule heat. The possible types of domains and their shapes are determined, and the current-voltage characteristics of the film is found with allowance for domain formation.

We consider a thin superconductor film on a dielectric substrate. The heat conduction equation in the film is

$$C\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2} + \rho(T, j)j^2 - \frac{H}{b}(T - T_0)\,, \qquad (1)$$

if the conditions $H \ll \kappa/b$, κ_s/w are satisfied. Here C, κ, and ρ are the specific heat, thermal conductivity, and resistivity of the film, b and w are the film thickness and width, $H(T - T_0)$ is the heat flux into the substrate, whose temperature T_0 is independent of the coordinates, and κ_s is the thermal conductivity of the substrate.

We use a phenomenological model in which ρ is a power-law function of the temperature, viz., $\rho = \rho_n\Theta[T - T_c(j)]$, where ρ_n is the resistivity in the normal state. The dependence of $T_c(j)$ on j is assumed by us to be given by the relation

$$T_c(j) = T_c[1 - (j/j_c)^{2/3}], \tag{2}$$

which follows from the Ginzburg-Landau theory.

The equation for the temperature, assuming it to be uniformly distributed,

$$T - T_0 = b\,\rho(T, j)j^2/H \tag{3}$$

has at $T_0 < T_c$, in general, three roots. The smallest root (T_0) corresponds to $\rho = 0$, and the largest ($T_2 = T_0 + b\rho_n j^2/H$) corresponds to $\rho = \rho_n$. The field in the film, corresponding to the middle root $T_1 = T_c(j)$, is obtained by determining $\rho(T, j)$ from (3) and using (2):

$$E = H[T_c(j) - T_0]/bj = \rho_n j_c(\tau - i^{2/3})/qi, \tag{4}$$

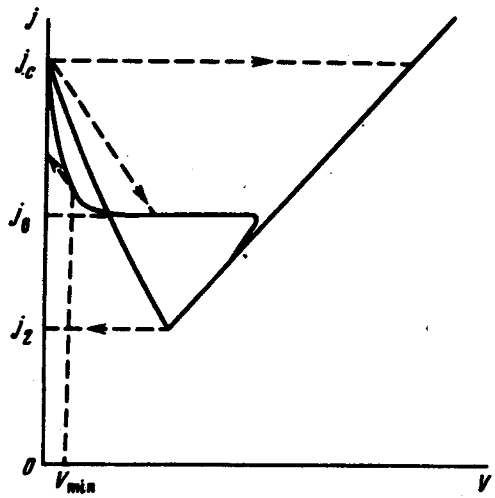
where $i = j/j_c$, $\tau = 1 - T_0/T_c$, and $q = \rho_n h j_c^2/HT_c$. Thus, the current-voltage characteristic (CVC) at a uniform temperature distribution is N-shaped [1], as shown in the figure. The minimal current on the descending section, j_2, is determined from the condition $T_2 = T_c(j_2)$ or, equivalently, from the condition $E = \rho_n j_2$, which takes the form $qi_2^2 + i_2^{2/3} - \tau = 0$. It follows that the N-shape of the CVC is sufficiently clearly pronounced, i.e., the current i_2 is noticeably lower than the critical $\tau^{3/2}$ if $q\tau^2 \gtrsim 1$.

From (1) and from the form of the CVC it follows that the superconductor film behaves like a semiconductor with N-shaped or S-shaped CVC [2]. Namely, at a given current the film goes over to the normal state at $j = j_c$ and back to the superconducting state at $j = j_2$ (the horizontal dashed lines with the arrows in the figure). On the other hand, if the voltage is given (the load resistance R_{ext} is small enough), then the homogeneous distribution of the temperature on the descending section of the CVC is unstable and an immobile temperature domain is produced in the film. The possible types and shapes of the domains can be determined from (1) by putting $\partial T/\partial t = 0$. An analysis (similar, e.g., to the analysis of the equations for Gunn domains), shows that there exist current densities j at which an "equilibrium" exists between the superconducting and normal sections of the films, each of which is larger than the characteristic length $\ell_T = (\kappa b/H)^{-1/2}$. The superconducting section is then at a temperature T_0, and the normal at $T_2(j_0)$. The current density j_0 satisfies the equation

$$\int_{T_0}^{T_2(j_0)} [j^2\rho(\eta) - H(\eta - T_0)/b]d\eta = 0$$

and the inequalities $j_2 < j_0 < j_c$.

At $j > j_0$ it is possible to have a temperature distribution in the form of a higher-temperature (normal) domain in a film with $T = T_0$, and at $j < j_0$ we can have a lower-temperature (superconducting) domain in a film with $T = T_2(j)$. As $j \to j_0$ these domains expans without limit.



CVC of superconducting film with uniform temperature distribution (thick line) and of film with temperature domain (thin line). The horizontal lines with arrows show jumps at large load resistance, and the inclined lines jumps at low load resistance.

In the model considered here, the shape of the domain and the shape of the CVC with a domain can be determined in explicit form. For the normal domain we have

$$\frac{T - T_0}{T_0} = \begin{cases} qi^2 - i[2q(i^2q/2 + i^{2/3} - \tau]^{1/2}\mathrm{ch}\left(\dfrac{x}{l_T}\right) & |x| < x_c \\ (\tau - i^{2/3})\exp[(x_c - |x|)/l_T] & |x| > x_c \end{cases}.$$

Here x_c is half the dimension of the normal region, i.e., the distance from the peak of the domain to the point at which $T = T_c(j)$. It is equal to

$$x_c = l_T \operatorname{arc\,ch} \frac{qi^2 + i^{2/3} - r}{i\,[\,2q(qi^2/2 + i^{2/3} - r)\,]^{1/2}}\,. \tag{5}$$

We see that x_c, and hence the width of the domain, increases like $\ln|j - j_0|$ as j approaches j_0, when the denominator of (5) vanishes.

Since only the normal section of width $2x_c$ contributes to the film resistance, the CVC of a film with a normal domain takes the form $V = 2\rho_n j x_c(j)$. Strictly speaking, this expression is valid if the domain is not close to the contacts. Otherwise it becomes necessary to take the boundary conditions into account. At definite boundary conditions, (e.g., $\partial T/\partial x = 0$ on the contacts), the CVC in the presence of the domain can be obtained by the procedure used in the case of semiconductors. It is shown in the figure. In the case of a dumbell-shaped film, it is reasonable to expect the normal domain to appear in the middle of the film, whereas the superconducting domains should appear on the boundary, and since the edge effects are significant in this case, the existence of such domains (and of the CVC section at $j < j_0$) is not so obvious (no decreased-field domains were observed in the Gunn effect, although their possible existence does follow from the model equations).

Thus, if R_{ext} is smaller than the normal resistance of the film, then a jump from the vertical section of the CVC (the inclined dashed line with the arrow) can occur, and a normal domain can be produced in the film. When the voltage is reduced, the current will remain practically constant (the broad domain becomes narrower) at $I_0 = j_0 S$, where S is the film cross section area, until the voltage decreases to the value V_{min}, given at $R_{ext} \gg R_d = \rho_n \ell_T/S$ by

$$V_{min} = I_0 R_d \ln\left[(R_{ext}/R_d)/2(1 + \frac{2}{3}q^{-1} i_0^{-4/3})\right].$$

At V_{min}, a break along the vertical occurs (see the figure). This is precisely the CVC observed in [5]. Estimates for an Sn film [5] at $H \simeq 2$ W/cm^{2}deg [6], $\kappa(\pi^2/3)\sigma(k^2T/\ell^2)$, $\sigma_\square \sim 5\Omega^{-1}$, $I_{c_2} \sim 0.1$ A, $T = 2°$K, $w \sim 0.1$ mm, and a length several times larger than w yield $\ell_T \sim 3\mu$ and $q\tau^2 \sim 0.3 - 1$.

Near the critical temperature, when $\xi \sim \ell_T$ at low voltages, it is possible in principle to observe the Josephson effect in a homogeneous film: the temperature domain is the N layer.

We note that the hysteresis of the CVC can be due in principle also to a non-thermal mechanism [7, 8]. The times required for a stationary CVC to be established in this case will apparently be much shorter than the thermal times.

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