SELF-FOCUSING OF LANGMUIR OSCILLATIONS

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Computer calculations and a qualitative analysis of the equations were used to investigate the character of nonstationary self-focusing of Langmuir oscillations of an isotropic plasma. The conditions under which self-focusing leads to effective plasmon energy dissipation are determined.

We report here the results of an investigation of the self-focusing of spatially-localized Langmuir oscillations of an isotropic plasma. The interest in this question is due in particular to a hypothesis advanced in [1], that collapse of Langmuir oscillation is possible and can lead to their effective dissipation.

We consider the evolution of a spherically-symmetrical distribution of longitudinal plasma oscillations with an electric field $\vec{E} = \vec{r}_0 E(r, t)\exp(i\omega_p t)$, where $\vec{r}_0$ is a unit vector. To describe the self-action, we use a standard system of equations consisting of the parabolic equation for the slow amplitude of the field E(r, t) and the equation for the small perturbations δn of the plasma density, which arise in an inhomogeneous high-frequency field

$$-2i\frac{\partial A}{\partial \tau} + \Delta_x A - \frac{2}{x^2}A - nA = 0, \qquad (1)$$

$$\frac{1}{u^2}\frac{\partial^2 n}{\partial \tau^2} - \Delta_x n = \Delta_x |A|^2. \qquad (2)$$

We have introduced here the dimensionless variables $x = \tau(3r_D\sqrt{g})^{-1}$, $\tau = \omega_p t/3g$,

$$A = E(16\pi N T_e/3g)^{-1/2},\ n = 3g\frac{\delta n}{N},\ g = \frac{T_e}{T_e + T_i}\frac{M}{mu^2},\ \Delta_x = \frac{1}{x^2}\frac{\partial}{\partial x}x^2\frac{\partial}{\partial x},$$

r_D is the Debye radius, T_e, T_i and m, M are the temperatures and masses of the electrons and ions, N is the unperturbed plasma concentration, and u is a scaling parameter.

An analysis of the system (1), (2) shows that one must distinguish between between two stages of self focusing of oscillations with total energy exceeding the critical value. At low amplitudes, $A_m^2 \ll u^2$, the field distribution varies slowly and the perturbations of the concentrations have time to be removed, at the speed of sound, from the field region, so that the nonlinearity of the medium is local, $n = -|A|^2$. The self-focusing process is analogous in this case to the process in nonlinear optics, and is modified somewhat because of the vector character of the field ($\vec{E} \equiv 0$ at r = 0) and because of the three-dimensional character of the problem. The value of the field at the maximum increases because of the simultaneous shift of the maximum to the center r = 0 and because of the reduction of its width. However, the "subsonic" self-focusing can lead only to an increase of the amplitude $|A|^2 \lesssim u^2$, which is equivalent to the relation $W/NT_e \lesssim m/M$ for the energy density. The main problem is therefore the character of the self-focusing of the oscillations during the succeeding "supersonic" stage, at which the rate of the change of the field exceeds the speed of sound.

To obtain the answer, we can attempt to simplify the problem by omitting from (2) the term that describes the transport of the perturbations[1)]

$$\frac{\partial^2 n}{\partial \tau^2} = u^2\Delta_x |A|^2. \qquad (3)$$

The conclusion drawn in [1] concerning the collapse was based on a self-similar solution obtained for the system (1), (3) and containing a singularity of the electric field. This solution was obtained in [1], however, by using the incorrect assumption that the phase of the field remains independent of the coordinate r during this stage of self-focusing.

It can be shown that the system (1), (3) has only self-similar solutions of the type

$$A(x,t) = \frac{\mathcal{E}(z)}{t_0 - t}, \quad n(x,t) = \frac{N(z)}{t_0 - t}, \quad z = x(t_0 - t)^{-1/2}, \tag{4}$$

but these solutions do not satisfy the total-energy conservation law that must be satisfied by the solution of the initial system with finite energy. Indeed, as $t \to t_0$ the total energy is given by $p \simeq 4\pi\int_0^\infty |A|^2 x^2 dx \sim (t - t_0)^{-1/2}\int_0^\infty |\varepsilon^2(z)| z^2 dz$. If such a self-similar solution does contain any information on the evolution of oscillations with finite energy, this means apparently that self-focusing of these oscillations can give rise only to singularities that contain zero energy and are therefore of no physical interest.

To ascertain the character of the self-focusing during the "supersonic" stage, we have solved the initial system (1), (2) with a computer, using the following initial conditions:

$$A(0,x) = A\frac{x}{a_0}\exp\left(-\frac{x^2}{2a_0^2}\right), \qquad n(0,x) = -A^2(0,x), \frac{\partial u}{\partial r}(0,x) = 0. \tag{5}$$

By way of illustration, Figs. 1 and 2 show the results of the calculations for the parameters corresponding to the "supersonic" conditions, namely $A_0 = 6$, $a_0 = 2$, and $u = 0.25$. Figure 1 shows the time dependence of the maximum values of E and n, while Figs. 2a and 2b show the $E(\tau)$ and $n(\tau)$ distributions for the instants of time $t = 0$, $t = 2.25$, and $t = 4.25$. It follows from the figures that since the rate of change of the field exceeds the speed of sound, the focusing of the oscillations is simultaneously accompanied by accumulation, at the center ($r = 0$), of particles carried away from the region of maximum field. The dielectric constant of the plasma becomes negative at the center, $\varepsilon = -n < 0$, and this prevents the field maximum from moving and causes it to stop. The self-focusing proceeds therefore just as in the one-dimensional case, i.e., it is accompanied only by a decrease of the effective width of the spherical layer of the oscillations. The growth of the amplitude then slows down and the inverse defocusing process sets in, while the maximum concentration perturbation attains by that instant of time a quasi-stationary value $n_{max} = -|A|^2_{max}$. On the whole, the picture is quite complicated. The focusing is accompanied by a break-up, due to sound generation[2)], of that field region which captures part of the plasma oscillations and carries them off to infinity.

To obtain approximate relations, we use the aberration-free approximation well known in linear optics [2], i.e., we assume that during the self-focusing process the shape of the initial distribution remains unchanged, and only its characteristic scale a changes with time.

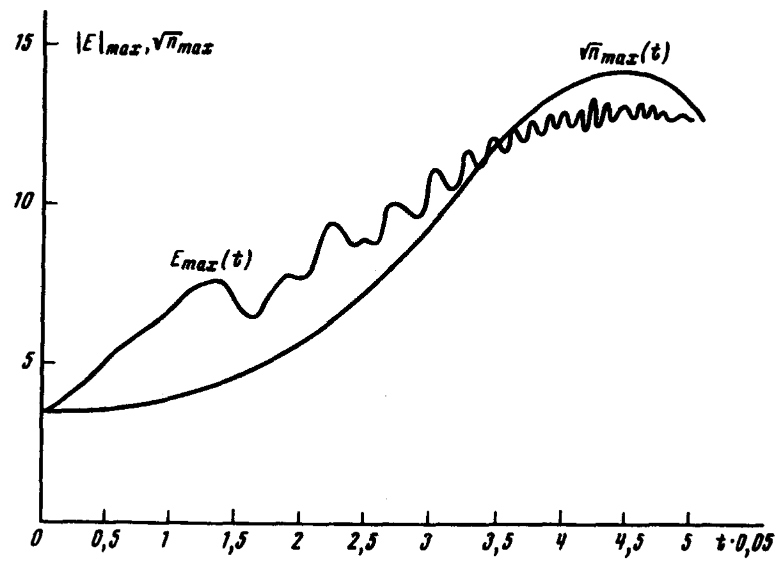

Fig. 1

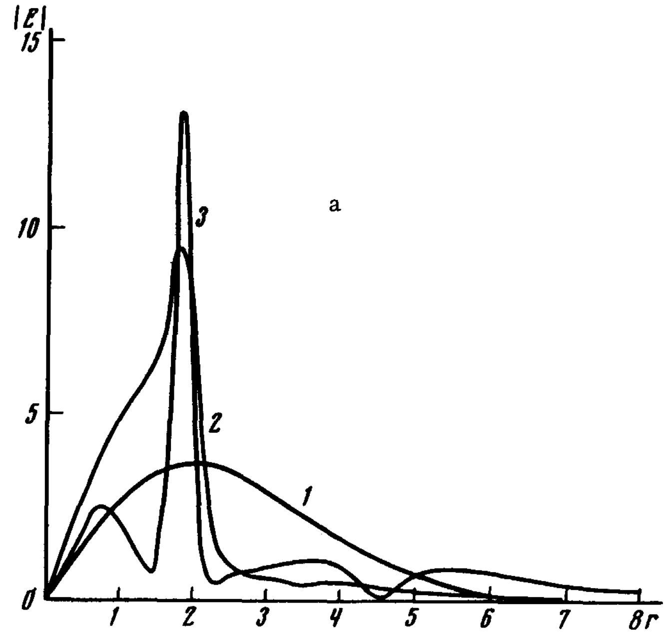

Fig. 2a

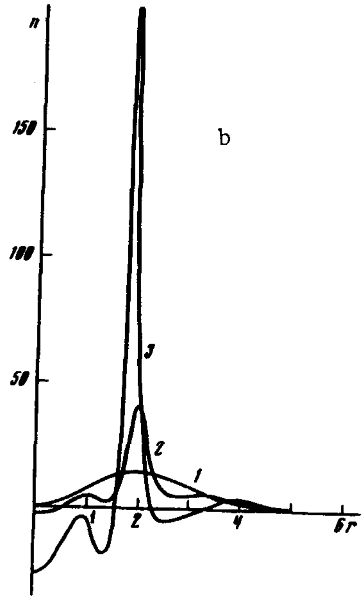

Fig. 2b

Without dwelling on the details (they will be published later), we note only that in accordance with the computer calculations we have considered two limiting cases. In the case of "subsonic" three-dimensional focusing of the initial field distribution (4), the scale a(t) decreases to a value $a^* \sim (1/2)P_0^{1/3}$, where $P_0 = (3\pi^{3/2}/2)A_0^2 a_0$ is the total initial energy, and the minimum dimension reached upon further one-dimensional focusing of the spherical layer of oscillations amounts to $a_{min} \sim P_0^{-1/3}$. On the other hand, if the initial parameters are "supersonic," then simultaneous compression takes place, to a dimension $a_{min} \sim 2/\pi^{3/2}A_0^2 a_0$. Obviously, these relations overestimate the self-focusing effect[3], in agreement with the results of numerical experiments in which a_{min} was 2 — 3 times larger the value obtained from the estimates.

We present, in dimensional units, the relations for the maximum wave number ($k_{max} = \pi/a_{min}$) reached in self-focusing of oscillations with "subsonic" and "supersonic" initial parameters, respectively

$$(k_{max} r_D) \sim \begin{cases} 1{,}7\left(\frac{W_0}{NT_e}\right)^{1/3}\left(\frac{m}{M}\right)^{2/3}(k_0 r_D)^{-1} \\ \frac{W_0}{NT_e}(k_0 r_D)^{-1} \end{cases} . \quad (5)$$

We have assumed here that $T_e \gg T_i$, $u \sim 1$ and W_0 and k_0 are the initial energy density and wave number of the oscillations. From this we can obtain, for example, the condition under which the self-focusing causes the oscillations to fall in the spectral region in which Landau damping plays an essential role ($k_{max} r_D \sim 1$): this takes place if

$$\frac{W_0}{nT_e} > \begin{cases} 0{,}1(k_0 r_D)^3\left(\frac{M}{m}\right)^2 \\ (k_0 r_D) \end{cases} . \quad (6)$$

It follows from (6), in particular, that oscillations satisfying the conditions of weak turbulence can in essense never reach the region of strong absorption as a result of self-focusing. In the absence of of collisions, the energy of such oscillations can apparently be dissipated only as a result of excitation of ion sound under quasiperiodic pulsations of the region of Langmuir oscillations.

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1) This approximation is of course valid in the time interval $\tau < a/u$, where a is the space scale.

2) The introduction of model-dependent sound damping into Eq. (2) leads to a weakening of this breakup.

3) To estimate the upper bound of the results of one-dimensional self-focusing, we assume $n = -|A^2(\tau)|$.

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[1] V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys.-JETP 35, 908 (1972)].
[2] V. I. Bespalov, A. G. Litvak, and V. I. Balanov, in: Nelineinaya optika (Nonlinear Optics), Nauka, 1968, p. 428.

RAMAN SCATTERING OF LIGHT BY SURFACE POLARITONS IN MEDIA WITH INVERSION CENTERS

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A broken symmetry method is proposed and makes it possible to use Raman scattering of light (RS) of first order by surface polariton to study the dispersion of the dielectric constant in media (crystals, glasses) having an inversion center. A formula is obtained for the intensity of the considered process, and the dependence of the line width of the RS by surface polaritons on the scattering angle is determined.

1. It is known that the Raman scattering (RS) of light by polaritons, in conjunction with lasers, has yielded valuable information on the dispersion of light in crystals. In centrosymmetrical media (CSM) such as crystals or glasses, however, this method cannot be used, for in these media the nonlinear susceptibility tensor $\chi_{ij\ell}$, which determines the intensity of the process, vanishes identically. The situation changes, however, if one resorts to RS of light by surface polaritons under conditions when at least one of the surfaces of the CSM borders on a medium that is transparent[1)] to the laser radiation, but one having no inversion center. Since the electromagnetic field in the surface polariton differs from zero over distances on the order of its wavelength ($\lambda \simeq 10\ \mu$) on both sides of the interface, and consequently also in the region where $\chi_{ij\ell} \neq 0$, the intensity of the RS by the surface polariton turns out to be different from zero and, as will be shown by the calculation below, is strong enough to be experimentally observable[2)]. This conclusion agrees also with the results of [1], where RS of light by surface polaritons was first observed in a medium without an inversion center (GaAs on a sapphire substrate). Naturally, in the RS spectrum obtained in [1] the most intense peaks corresponded to excitation of volume polaritons, and this hindered the observation of the effect. In the "inverted" situation proposed by us, where absence of an inversion center is ensured by the choice of the substrate, the volume polaritons of the CSM should not be excited at all.

2. We proceed to calculate the effectiveness of the process. Neglecting the possible anisotropy of the media, and also the dispersion of light in the substrate, we assume its dielectric constant ε_1 to be positive, and assume $\chi_{ij\ell} = \chi|e_{ij\ell}|$, where $e_{ij\ell}$ is a fully antisymmetric tensor of third rank. The measured dispersion of the surface polariton, $\omega = \omega(\vec{k})$, is determined (see [2]) by the relation $k^2 = \omega^2\varepsilon_1\varepsilon/c^2(\varepsilon + \varepsilon_1)$, where $\varepsilon = \varepsilon(\omega) < 0$ is the dielectric constant of the CSM. By way of theperturbation we use the operator

$$\hat{H} = -\frac{1}{2}\int_{z>0} \chi_{ijl}\hat{E}_i\hat{E}_j\hat{E}_l^p d\mathbf{r},$$

where $\hat{\vec{E}}$ and $\hat{\vec{E}}^p$ are the electric intensity operators of the high-frequency field and of the field in the surface polariton. Neglecting the difference between the refractive indices of the laser