RELATIVISTIC EXPANSION OF AMPLITUDES AND THE PARTON MODEL

Ya. A. Smorodinskii
Joint Institute for Nuclear Research
Submitted 29 October 1973
ZhETF Pis. Red. 19, No. 1, 55 — 57 (5 January 1974)

It is shown that it is convenient to describe the properties of the parton model in a 4-dimensional cylindrical coordinate system. The connection between the relativistic numbers and the concept of relative coordinates is elucidated.

The description of electronic inclusive reactions in the parton model, after Feynman [1], is based on four hypotheses: 1) the Bjorken scaling, with the cross section described as a function of one variable, $x^{-1} = -2M\nu/q^2$, where M is the proton mass, ν is the energy lost by the electron, and q^2 is the square of the transferred momentum; 2) the transferred momentum is absorbed by the (velocity) spectrum that can be simulated by the spectrum of free particles (partons); 3) the parton spectrum takes the form of a plateau if the independent variable is assumed to be the rapidity; 4) the spectrum of the final particles is a replica of the parton spectrum.

Such a description, however, raises difficulties connected with the space-time analysis of the process.

It is useful to trace the evolution of these properties by describing the distribution functions in terms of the relativistically-invariant expansions investigated in [2]. We use here a coordinate system with a symmetry that corresponds to the symmetry of the problem, the so-called C-system[1)].

We choose the variable to be the relative relativistic 4-velocity u ($u^2 = 1$). We introduce the following parametrization:

$$u_0 = \mathrm{ch}\,\zeta\,\mathrm{ch}\,\eta, \qquad u_2 = \mathrm{sh}\,\zeta\cos\phi\,,$$
$$u_3 = \mathrm{ch}\,\zeta\,\mathrm{sh}\,\eta, \qquad u_1 = \mathrm{sh}\,\zeta\sin\phi.$$

We see immediately that

$$\eta = \frac{1}{2}\ln\frac{u_0 + u_3}{u_0 - u_3}\,,$$

and that the longitudinal rapidity m coshζ plays the role of the "transverse" mass

$$m\,\mathrm{ch}\,\zeta = m_\perp = (\epsilon^2 - p_\parallel^2)^{1/2}\,.$$

It is shown in [2] that in such a coordinate frame, a complete orthogonal system of functions is made up of the system of solutions of the d'Alambert equations, in the form

$$\Psi_m(p,\tau) = N_m(p,\tau)\,e^{im\phi}e^{i\tau\eta}F\left(\frac{1+i\tau+ip}{2},\frac{1-i\tau-ip}{2};\, m+1;\, \mathrm{th}^2\frac{\zeta}{2}\right).$$

A state is characterized by three quantum numbers: one the ordinary magnetic m, and two relativistic, τ and p, which are connected with the square of four-dimensional angular momentum of the system. The normalization factor $N_m(p, \tau)$ is given in [2]. We do not need it here.

The expansion of the distribution function of the partons or the secondary particles contains a Fourier expansion in the longitudinal rapidity. We use precisely this fact. It can also be noted that the Lorentz transformation along the particle motion reduces simply to the substitution $\eta \to \eta + \eta^*$, where η^* is the rapidity of the new reference frame; this reflects the additivity of the rapidities.

If we expand the spectrum in terms of the functions introduced in this manner, then obviously an effective part will be played in the expansion by a "frequency" band $\Delta\tau$, the order of magnitude of which is

$$\Delta\tau \sim 1/\Delta\eta ,$$

where $\Delta\eta$ is the width of the plateau in the distribution. As $\Delta\eta \to \infty$, which corresponds to infinitely large energies, we have $\Delta\tau \to 0$ and even $\tau \to 0$. It is clear that a distribution independent of the rapidities corresponds to the zeroth Fourier component.

The formation of the secondary-particle spectrum takes place in the parton model with preservation of this property at infinitely high energies, and it can be described as a process with an asymptotic conservation of the quantum number τ.

We can now raise the question of the physical meaning of the number τ. Since τ is conjugate to the component of the relative 4-velocity, we can regard τ, in analogy with an ordinary plane wave, as a relative coordinate in a certain quasicoordinate space. We use the word "quasicoordinate" since such soordinates cannot be derived directly from the coordinates that are the arguments of the wave functions of the system.

The use of relative coordinates in a relativistic system of two particles was proposed by Kadyshevskii [3].

We can thus state that as $\eta \to \infty$ the spectra of the partons and of the secondary particles are described by a delta function $\delta(\tau)$, and that at finite energies the delta function is replaced by a narrow distribution with respect to τ.

The appearance of a delta function in such a picture can be interpreted as a description of a contact interaction in a space introduced in this manner. A similar picture arises, as is well known, in the inverse nonrelativistic limiting case, as the wavelength $\lambda \to \infty$. In this case we get isotropy with respect to the angle, and the quantum number ℓ vanishes. The analogy $(\lambda \to \infty, \ell \to 0)$ and $(\eta \to \infty, \tau \to 0)$ seems to be not devoid of meaning.

The foregoing reasoning shows that the space of the coordinates τ (and accordingly p) can indeed be convenient for the description of processes at high energies.

In conclusion, I wish to thank I. S. Shapiro for the opportunity of reading his paper, in which similar ideas are developed, and V. G. Kadyshevskii for a stimulating discussion.

[1)] We note that this system was used in an old but still valuable paper by G. A. Milekhin [4].

[1] R. Feynman, High Energy Collisions, 3-rd Int. Conf. at Stony Brook, 1969, p. 237.
[2] N. Ya. Vilenkin and Ya. A. Smorodinskii, Zh. Eksp. Teor. Fiz. 46, 1793 (1964) [Sov. Phys.-JETP 19, 1209 (1964)].
[3] V. G. Kadyshevskii, R. M. Mir-Kasimov, and N. B. Skachkov, Elem. chast. atom. yad. 2, 636 (1972) [Sov. J. Part. Nuc. 2, 69 (1973)].
[4] T. A. Milekhin, Zh. Eksp. Teor. Fiz. 35, 1185 (1958) [Sov. Phys.-JETP 8, 829 (1959)].