Nucleus	ζ	R_c, F	E_t, MeV	σ_0, b
Th	1,314	43,5	530	30,8
U	1,343	51,3	475	53,8
Pu	1,372	59,8	425	91,0
Cm	1,401	68,7	385	146,0
Cf	1,431	78,0	355	225,0
Fm	1,460	88,0	330	341,0

1) The possibility of production of electron-positron pairs from vacuum in a strong electric field was predicted long ago in quantum electrodynamics, but this effect was not yet observed experimentally. The spontaneous production of e^+ in a Coulomb field with charge $Z > Z_c$ is of interest as a check on the Dirac equation in strong external field and as a check on the properties of physical vacuum [1, 4], and also from the point of view of verifying the linearity of the fundamental equations of quantum electrodynamics.

2) Since the charge of each of the nuclei is Z < 137, there is no "falling to the center" in a Coulomb field $-Z\alpha/r$, and the nuclei can be regarded as pointlike. The correction for the finite dimensions of the nucleus at Z ≈ 90 to 100 increases the energy of the principal term by only $\Delta\varepsilon \sim 1.5\times10^{-3}$ < 1 keV (see formula (12) of [9]).

3) Repeated indices mean summation. The system (7) has a solution satisfying the boundary conditions (exponential decrease at infinity and smallest singularity as $x \to 0$) exists only for discrete values of R, if $\zeta = 2Z/137$ is fixed. The largest of these three roots determines the $R_c(\zeta)$ dependence for the main term.

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DEEP INELASTIC LEPTON-PROTON SCATTERING AND μ-e UNIVERSALITY

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To verify μ-e universality, a joint analysis was performed of the data on deep inelastic μ-p and e-p scattering. It is shown that these data are compatible if the μ-p-scattering cross sections are renormalized.

We report here the results of a joint analysis of the data on deep inelastic μ-p scattering, obtained in [1], and the data of the SLAC-MIT group [2, 3] on deep inelastic e-p scattering. The main purpose of the analysis was to check on the μ-e universality.

The data of [1] on μ-p scattering were obtained at a muon momentum 12 GeV/c and $q^2 \leq 4$ $(\mathrm{GeV/c})^2$. The e-p scattering cross sections were measured [2, 3] at electron energies up to

20 GeV and 0.25 $(\text{GeV}/c)^2 \leq q^2 \leq 19.72\ (\text{GeV}/c)^2$. To compare the cross sections of deep inelastic μ-p and e-p scattering, the data of [2, 3] were extrapolated in [1] to the region of the μ-p data of [1].

We use in this paper another method of comparing the cross sections of deep inelastic μ-p and e-p scattering.

The cross sections for the scattering of protons by leptons, if the lepton mass can be neglected, is given by (in the lab)

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{a^2}{4E^2 \sin^4\theta/2} \cos^2\theta/2\,[W_2 + 2W_1 \operatorname{tg}^2\theta/2]\,. \tag{1}$$

Here E and E' are the initial and final energy of the lepton, θ is the lepton scattering angle, and the quantities W_1 and W_2 characterize the hadronic part of the process and depend in the general case on the scalars q^2 and $\nu = E - E'$. The functions $2MW_1$ and νW_2 are connected by the following general relation

$$2MW_1 = \omega\,\nu W_2 \frac{1 + q^2/\nu^2}{1 + R}\,. \tag{2}$$

Here s and T are the total cross sections of a proton absorbing a virtual photon having longitudinal and transverse polarization, respectively.

$$\omega = 2M\nu/q^2 \tag{3}$$

$$R = \sigma_s/\sigma_T\,, \tag{4}$$

where σ_S and σ_T are the total cross sections of a proton absorbing a virtual photon having longitudinal and transverse polarization, respectively.

In [4] we analyzed all the available data on deep inelastic e-p scattering. It was shown that in the region $W \geq 2.3$ GeV (W is the mass of the final hadron system) the data are well described if one assumed for νW_2 the expression

$$\nu W_2 = \sum_{i=0} a_i \left(1 - \frac{1}{\omega}\right)^{i+3}. \tag{5}$$

The data can be described at different parametrizations R. We considered the following expressions corresponding to different models:

$$R = c_1 q^2/M^2 \tag{6}$$

$$R = c_2 q^2/W^2, \tag{7}$$

$$R = c_3 q^2/2M\nu, \tag{8}$$

$$R = \text{const}. \tag{9}$$

The results of the experiments on e-p scattering are also adequately described if one assumes the validity of the expression

$$2MW_1 = \omega\,\nu W_2 (1 + c_4/\omega)^{-1} \tag{10}$$

which coincides with the Callan-Gross equation [5] at sufficiently large ω. It was shown that it suffices to regard a and a as different from zero in cases (6), (7), (8), and (10), and a_0, a_1, a_2 different from zero at constant R.

A joint analysis of the data on μ-p and e-p scattering was carried out with the same parametrization of the structure functions as in [4]. The parameters were obtained by minimizing the functional

$$\chi^2 = \sum_k \sum_i \frac{1}{\Delta_{ik}^2} (\sigma_{ik}^{\text{exp}} - N_k \sigma_i^{\text{theor}})^2\,, \tag{11}$$

where σ_{ik}^{exp} is the differential cross section measured in the k-th experiment at the i-th point,

Results of joint analysis of the data on deep inelastic
μ-p scattering [1] and e-p scattering [2, 3]

	a_0	a_1	a_2		$\chi^2/\overline{\chi^2}$	N_μ
$R = c_1 q^2/M^2$	1,62 ±0.02	–	-1,48 ±0,02	c_1 = 0,038 ±0,004	188/182	0,840 ±0,017
$R = c_2 q^2/W^2$	1,64 ±0,02	–	-1,50 ±0,02	c_2 = 0,460 ±0,050	204/182	0,850 ±0,017
$R = c_3 q^2/2M\nu$	1,65 ±0,02	–	-1,51 ± 0,02	c_3 = 0,900 ±0,090	204/182	0,838 ±0,016
R = const	1,22 ±0,06	0,99 ±0.16	-2,07 ±0,10	R = 0,230 ±0,030	223/181	0,827 ±0,017
$2MW_1 = \omega\nu W_2(1 + c_4/\omega)^{-1}$	1,66 ±0,02	—	-1,53 ±0,02	c_4 = 0,690 ±0,080	222/182	0,838 ±0,016
$2MW_1 = \omega\nu W_2(1 + c_4/\omega)^{-1}$	1,64 ±0,02	—	-1,50 ±0,02	c_4 = 0,630 ±0,090	162/150	–

$\Delta_{i,k}$ is the error of σ_{ik}^{exp}, and σ_i^{theor} is the cross section given by expression (1), while N_k are norms. We assume $N_e = 1$ and N_μ to be a variable parameter. The parameters obtained by us are listed in the table. The figure shows a plot of the function νW_2 against ω in the case when the structure functions are connected by relation (10).

As a result of the joint analysis of the data of [1 - 3] we arrive at the following conclusions:

1. The data obtained in [1] on deep inelastic μ-p scattering are compatible with the data on deep inelastic e-p scattering in [2, 3] if the μ-p data are renormalized.

2. The norm N_μ does not depend (within the limits of errors) on the parametrization of R.[1)]

3. The values of the coefficients a_i and c_i, obtained from the joint analysis of the μ-p and e-p data, coincide within the limits of errors with the values of the corresponding coefficients obtained in [4] by analysis of e-p scattering data (the last line of the table lists the values of the parameters obtained from the analysis of e-p data).

4. The difference between the cross sections of deep inelastic μ-p and e-p scattering is apparently connected with systematic errors and does not indicate a deviation from μ-e universality.

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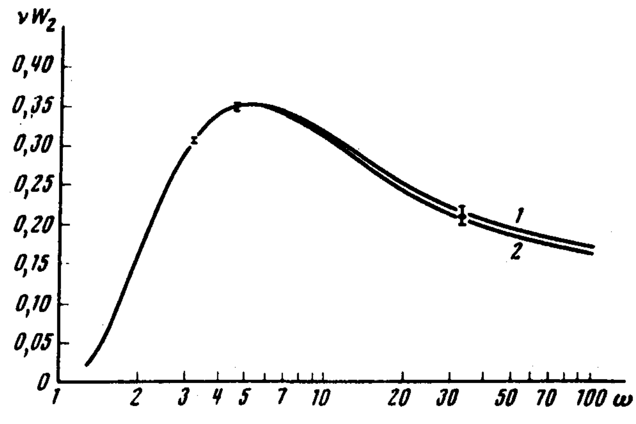

The function of νW_2. Curve 1 was obtained from an analysis of e-p and μ-p data, and curve 2 from e-p data.

[1)] We note that the values obtained by us for the norm N_μ differs from the corresponding value obtained in [1] by extrapolation.

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