COOLING WITH THE AID OF HIGH-FREQUENCY ENERGY

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We consider the feasibility in principle of cooling a two-level system (e.g., spins in a magnetic field) by high-frequency energy pumping (a generalization of the known method of dynamic orientation of spins) and the possibility of making use of quasienergy to select the conditions under which the process can be realized.

According to the second law of thermodynamics, a small amount of energy Q_1 is taken in a refrigerator from a system having a low temperature T_1, a high energy $Q_2 = Q_1 + A$ is transferred to a thermostat having a higher temperature T_2, and the work A is performed by an external energy source; the maximum cooling is given by the expression $T_1 \geq T_2Q_1/Q_2$.

Assumed that the cooled system is a two-level one, with an energy difference δ between the two states A and B, i.e., $\delta = E_B - E_A$. Let the thermostat contain another two-level system (C, D) with an energy difference $\Delta = \delta + \hbar\omega = E_D - E_C$ (see Fig. 1). It is natural to assume that application of a high-frequency energy source of frequency ω makes it possible to draw adiabatically an energy δ from the system (A, B) and give up an energy $\Delta = \delta + \hbar\omega$ to the system (C, D), drawing the difference $\hbar\omega$ from an external energy source. It is then possible to cool (A, B) to a temperature T_1 lower than the thermostat temperature T_2, to the limit

$$T_1 = T_2\delta/(\delta + \hbar\omega). \tag{1}$$

In a two-level system, the temperature is determined by the ratio of the populations of the two levels[1])

$$[B]/[A] = \exp(-\delta/T_1),$$
$$[D]/[C] = \exp(-\Delta/T_2). \tag{2}$$

The thermodynamic relation between T_1 and T_2 corresponds to the assumption that transitions $B + C \rightleftarrows A + D$ become possible in the presence of the high-frequency field

$$\frac{d[B]}{dt} = -\frac{d[A]}{dt} = K_1[A][D] - K_2[B][C]. \tag{3}$$

It is also assumed that $K_1 \equiv K_2 = K$, and the probability of both transitions is proportional to the intensity of the high-frequency pump (or at least vanishes without the pump and depends in like manner on the pump[2])). In this case we obtain in the stationary state $d[b]/dt = 0$

$$[A][D] = [B][C], \tag{4}$$

which yields

$$[B]/[A] = [D]/[C], \quad T_1 = T_2\delta/\Delta.$$

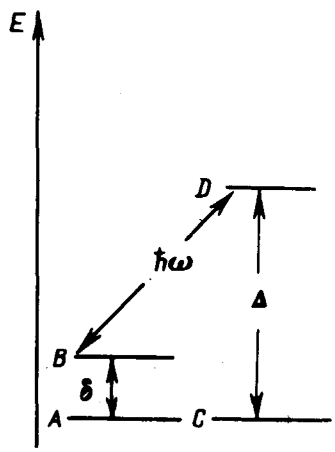

Fig. 1

This is the situation to which, in particular, dynamic polarization of nuclei [1] reduces in the limit of weak coupling between the nucleus and the electron.

To prove that such a process is possible, and also to deliberately choose the suitable conditions for the process, we use the quasienergy concept [2 - 5].

We imagine the catalyst of the process (3) to be a third two-level system (F, G), in which, as a result of the joint action of the high-frequency pum and the constant fields (magnetic field, the lattice field, the spin-orbit or the hyperfine splitting, etc.) there is a quasilevel difference δ

$$E_G^{(0)} - E_F^{(0)} = \delta. \tag{5}$$

But according to the general theory the quasienergy is defined in modulo $h\omega$, and each of the states of (F, G) has satellites (n_1, n_2, n_3 are integers)

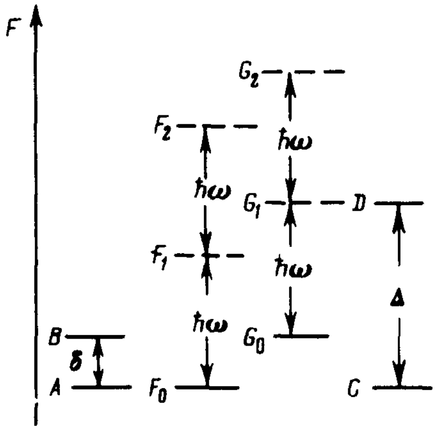

Fig. 2

$$E_G^{(n_1)} - E_G^{(n_2)} = \delta + n_3\hbar\omega, \quad n_3 = n_2 - n_1. \tag{6}$$

Thus, the system (F, G) is at resonance simultaneously with (A, B) and (C, D) (with the former as the ground-state transition, with the latter as a result of the satellite), see Fig. 2. It is important that the ground state and the satellites, e.g., F_0, F_1, F_2, F_{-1}, F_{-2} form toegher a single complex F - a quasienergy state, within which the ratios of the amplitudes and of the phases are maintained rapidly and without dissipation by the high-frequency field. There exists a population [F], but [F_0], [F_1], etc. cannot exist as independent units.

We consider the resonant transitions

$$A + G \rightleftarrows B + F; \qquad C + G \rightleftarrows D + F.$$

From the detailed balancing principle (from the equality of the matrix elements for the direct and inverse transitions), it follows that in the stationary state

$$\frac{[B]}{[A]} = \frac{[G]}{[F]} = \frac{[D]}{[C]} \tag{7}$$

thus proving the statements of the first part of the article. We assume for simplicity that (A, B) and (C, D) are not split by the pump, making the determination of their temperature unique[3]. In principle one cannot exclude a direct utilization (for radio reception, etc.) of F_0 and G_0 themselves.

An important role is played in the process by the assumption that the resonant processes predominate. A nonresonant process C + F = G + D, with part of the energy drawn from the lattice, is possible, but this would spoil all the derived relations; we neglect processes of this type.

The probability of the process C + G → F + D includes the weight of the satellite G_1 in the state G; accordingly, this probability tends to zero when the pump amplitude is increased. But the same weight G_1 is contained also in the probability of the inverse process F + D → C + G. Therefore, neglecting losses, the cooling limit (as follows also from (1)) does not depend on the pump amplitude, which influences only the power of the refrigerator.

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1) We designate the populations by square brackets, and all statistical weights are assumed for simplicity to be equal to unity.

2) We neglect here dissipative interactions between the system (A, B) and the thermostat. The pump is assumed intense enough to be treated classically, so that in the exact quantum expressions the probabilities of the absorption ($\sim n$) and of emission (spontaneous + induced, $\sim 1 - n$) of a quantum $\hbar\omega$ we can neglect unity in comparison with n.

3) For the quasienergy system (F, G) the populations and their ratios are determined uniquely, but the effective temperature is not uniquely defined, owing to the non-uniqueness of the quasienergy itself, $T_{eff}^{(n)} = (\delta + n\hbar\omega)\{-\ln[F]/[G]\}^{-1}$.

[1] C. D. Jeffries, Dynamic Nuclear Orientation, Interscience, 1963.
[2] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 776 (1969) [Sov. Phys.-JETP 19, 529 (1969)].
[3] V. I. Ritus, ibid. 51, 1544 (1966) [24, 1041 (1967)].
[4] Ya. B. Zel'dovich, ibid. 51, 1492 (1966) [24, 1006 (1967)].
[5] Ya. B. Zel'dovich, Usp. Fiz. Nauk 110, 139 (1973) [Sov. Phys.-Usp. 16, No.3 (1973)].