CONTRIBUTION TO THE NONLINEAR THEORY OF CYCLOTRON INSTABILITY

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Frequency simulation methods are used to investigate the nonlinear dynamics of the cyclotron instability that sets in in the presence of a current flowing in a plasma in a direction perpendicular to the magnetic field. It is shown that ion capture leads to stabilization of the instability. The maximum electric-field amplitude of the excited wave is determined.

Ion currents flowing perpeudicular to a magnetic field produce in a plasma an electrostatic instability at harmonics of the electron cyclotron frequency [1]. An investigation of the nonlinear dynamics of this instability is very important for the explanation of the mechanisms of turbulent heating in experiments with collisionless shock waves and in θ pinches [2].

One of the possible mechanisms of stabilizing cyclotron instability is connected with the electronic nonlinearity, namely the presence of unstable oscillations leads to the scattering of the electrons by the turbulent pulsations of the electric field, and the oscillations stop growing if the effective collision frequency is high enough [3].

Another mechanism that stabilizes the cyclotron instability is connected with the deformation of the distribution function of the resonant ions as a result of the reaction of the excited oscillations [4].

The purpose of our investigations was to examine these stabilization mechanisms in the single-mode regime by using partial simulation with a computer [5]. In this case, the mechanism of the electronic nonlinearity causes the discrete electronic resonances $\delta(\omega - n\omega_{He})$, which occur if the wave propagates strictly perpendicularly to the magnetic field, to broaden by an amount $\sim 1/k\sqrt{e\phi_0/m_e}$ (ϕ_0 is the amplitude of the excited-wave potential), and at sufficiently large wave amplitude, when

$$\phi_0 \sim \frac{m_e}{e}\,\frac{\omega_{He}^2}{k^2} \tag{1}$$

(ω_{He} is the electron cyclotron frequency and k is the wave number of the excited wave), these resonances overlap and the cyclotron harmonics are nonlinearly damped by the electrons; this leads to stabilization[1)].

For an instability with a sufficiently small increment $\gamma < (m_e/m_i)^{1/2}\omega_{He}$ (γ is the wave growth increment), a more substantial stabilization mechanism is the capture of the ions in the potential wells produced by the wave excited by the ions, at an amplitude

$$\phi_0 \sim \frac{m_i}{e}\,\frac{\gamma^2}{k^2}\ . \tag{2}$$

The captured ions, executing phase oscillations in the potential well, do not exchange energy with the wave when averaged over the period of the oscillation, and the instability is thus stabilized.

We present here results of an investigation of this stabilization mechanism with the aid of a computer. We considered the kinetic instability that sets in at sufficiently large velocity spread of the resonant ions, $v_{Ti} \gtrsim \gamma/k$ (v_{Ti} is the ion thermal velocity). The ion current excites in the plasma a potential wave of frequency $\omega = n\omega_{He}$, corresponding to the maximum growth increment [4]

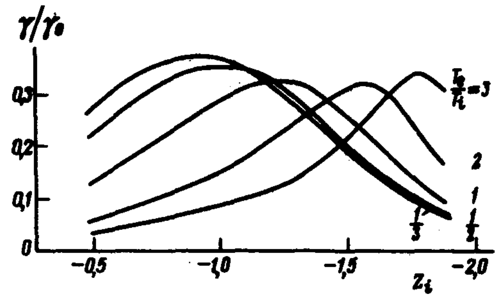

Fig. 1

$$\gamma = -\gamma_0 \frac{z_i e^{-z_i^2}}{\left[1 + \frac{T_e}{T_i}\frac{\psi(z_i)}{1+k^2\lambda_D^2}\right]^2 + \left[\frac{T_e}{T_i}\frac{\sqrt{\pi} z_i e^{-z_i^2}}{1+k^2\lambda_D^2}\right]^2}, \tag{3}$$

where

$$\gamma_0 = \frac{v_d}{v_{T_e}} \frac{T_e}{T_i} \omega_{Hc} \frac{1}{(1+k^2\lambda_D^2)^2}$$

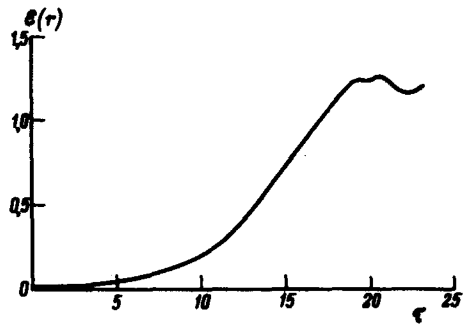

Fig. 2

T_e and T_i are respectively the temperature of the electrons and ions, $\lambda_D^2 = T_e/4\pi e^2 n_0$, $z_i = \omega - kv_d/kv_{Ti}$. v_d is the ion drift velocity relative to the electrons, and

$$\psi(z_i) = 1 - 2z_i e^{-z_i^2} \int_0^{z_i} e^{t^2} dt.$$

The dependence of the dimensionless growth increment γ/γ_0 on the detuning z_i at different values of the parameter T_e/T_i is shown in Fig. 1. We see that with increasing parameter T_e/T_i the maximum value of the increment shifts towards larger $|z_i|$, remaining approximately constant in magnitude, $\gamma_{max} \sim 0.3$. The presence of a maximum in the growth increment justifies the possibility of regarding the instability in the single-mode regime, since it is the oscillations with the maximum increment that will build up in the main. Accordingly, we express the electric field of the wave in the form

$$E(t,x) = E(t)\cos(kx - n\omega_{Hc}t + \alpha(t)). \tag{4}$$

In the dimensionless variables

$$\zeta = \frac{kx'}{2\pi}, \quad \nu = \frac{kv'}{2\pi\gamma_0}, \quad \tau = \gamma_0 t, \quad \epsilon = \frac{ekE}{m_i\gamma_0^2} \tag{5}$$

($\varepsilon \sim 1$ corresponds to the amplitude of the potential of the wave (2)), the initial system of equations for the problem (the equations of motion of the resonant ions and the equations for the amplitude and phase of the field) takes the form

$$\frac{d\zeta}{d\tau} = \nu, \quad \frac{d\nu}{d\tau} = \frac{\epsilon}{2\pi}\cos(2\pi\zeta + \alpha), \tag{6}$$

$$\frac{d\epsilon}{d\tau} = -\frac{8\pi^{3/2}}{1+k^2\lambda_D^2}\frac{T_e}{T_i}\frac{1}{\delta}\int_{-1/2}^{1/2}\int_{-\nu_0^m}^{\nu_0^m} \nu\cos(2\pi\zeta+\alpha)\exp\{-(\nu_0\delta + z_i)^2\} \times d\zeta_0 d\nu_0, \tag{7}$$

$$\epsilon\frac{d\alpha}{d\tau} = -\frac{1+k^2\lambda_D^2}{\sqrt{\pi}}\frac{T_i}{T_e}\epsilon + \frac{8\pi^{3/2}}{1+k^2\lambda_D^2}\frac{T_e}{T_i}\int_{-1/2}^{1/2}\int_{-\nu_0^m}^{\nu_0^m} \nu\sin(2\pi\zeta+\alpha)\exp \times \{-(\nu_0\delta + z_i)^2\} d\zeta_0 d\nu_0. \tag{8}$$

In these equations, the parameter $\delta = 2\pi\gamma_0/kv_{Ti}$ characterizes the degree of smearing of the ion flux, $\nu^m = kv^m/2\pi\gamma$, and v^m is the upper limit of integration with respect to velocity. Relations (6) - (8) have been written out in the reference frame of the wave, with $v' = v - v_{ph}$, $x' = x - v_{ph}t$, x_0 and v_0 are the initial values of the dimensionless coordinate and velocity of the particle, which is situated at the instant of time t at the point (x, v) of phase space.

The system (6) - (8) was integrated with a BESM-6 computer at $k^2\lambda_D^2 \ll 1$, $\delta = 1$, and $T_e/T_i = 1/2$ (the corresponding value is $z_i = -1.02$). We integrated the trajectories of 6×10^3 particles, the initial coordinates of which were varied in the range $-1/2 < \zeta_0 < 1/2$ in steps of $\Delta\zeta_0 = 1/20$, and the velocities were varied in the range $-3 < \nu_0 < 3$ with steps of $\Delta\nu_0 = 1/48$ and with time intervals $h = 5\times10^{-3}$. Figure 2 shows the results of the integration of Eqs. (6) - (8), namely the time dependence of the dimensionless field amplitude. The initial section at $\tau \leq 15$ corresponds to an exponential growth of the amplitude with increment ~ 0.3, which agrees with sufficient accuracy with the increment of the linear theory. At $\tau \simeq 20$, the maximum value $\varepsilon_{max} \sim 1.2$ is reached, followed by amplitude oscillations corresponding to the phase oscillations of the captured ions.

The energy of the wave excited in the plasma is given by

$$\frac{E^2}{8\pi} = \epsilon^2 \frac{n_0 m_i v_d^2}{2} \frac{\omega_{He}^2}{\omega_{oi}^2} \frac{\omega_{Hc}^2}{k^2 v_{T_e}^2} \left(\frac{T_e}{T_i}\right)^4 \frac{v_d^2}{v_{T_e}^2} . \tag{9}$$

A separate communication will be devoted to the derivation of Eqs. (7) and (8), to a more detailed examination of the stabilization by ion capture, and also to an investigation of the other stabilization mechanism connected with the nonlinearity of the electrons.

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1) Nonlinear damping of plasma oscillations propagating perpendicular to H_0 was investigated in [6].

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