DIFFRACTION LIMIT OF FOCUSING OF ULTRASHORT LIGHT PULSES BY A ZONE PLATE

V. N. Lugovoi
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 17 December 1973
ZhETF Pis. Red. 19, No. 3, 176 – 179 (5 February 1974)

An expression is derived for the field of a light pulse of arbitrary initial shape, with Gaussian initial intensity distribution over the cross section, passing through a zone plate. The variation of the time-dependent shape and of the duration of this pulse in the region of the zone plate is investigated. It is indicated on the basis of this investigation that it is possible to estimate the duration of sufficiently short light pulses.

As is well known, the passage of a parallel light beam that is stationary in time through a zone plate (say a system of concentric alternating transparent and opaque annular zone - Fresnel zones) is similar to the passage of this beam through a focusing (defocusing) lens. We consider below the passage of light pulses through a zone plate, and show that if these pulses are of sufficiently short duration the focusing by a zone plate differs in principle from focusing by a lens. The distinguishing features of focusing by a zone plate can be used to estimate the duration of an ultrashort light pulse.

We consider for concreteness the passage of a parallel linearly-polarized Gaussian beam with a time-dependent envelope f(t) and phase $\phi(t)$ through a plate in which the transitions between the transparent and opaque zones are gradual; the plate is located in the plane z = 0. In this case, the boundary condition at z = 0 for the transverse component of the electric field can be written in the form

$$\mathcal{E}_{z=0} = f(t)\, e^{-\frac{r_\perp^2}{2a_0^2}} \left[1 + m\cos\left(\frac{kr_\perp^2}{2R}\right)\right] \cos[\Omega t - \phi(t)]. \tag{1}$$

Here $r_\perp = \sqrt{x^2 + y^2}$ is the deviation from the beam axis, a_0 is the initial radius of this beam, $k = \Omega/c = 2\pi/\lambda$, c is the speed of light in the considered medium, Ω is the "central" frequency of the field oscillations in the beam, λ is the wavelength corresponding to this frequency in the medium, and the parameter R > 0 determines the position of the focal spot on the z axis. We start from the equation

$$\Delta\mathcal{E} - \frac{1}{c^2}\frac{\partial^2\mathcal{E}}{\partial t^2} = 0 \tag{2}$$

(in the region z > 0), with boundary condition (1) and with the radiation condition at $\sqrt{z^2 + r_\perp^2} \to \infty$. We confine ourselves here to the case of greatest practical interest

$$\lambda \ll a_0 \ll R. \tag{3}$$

Leaving out the intermediate calculations, we present for $\mathcal{E}$ an expression that is valid under the conditions (3)

$$\begin{aligned} &\mathcal{E} = \mathrm{Re}E, \quad E = E^+ + E^\circ + E^-, \\ &E^+ \equiv E^+(R) = \frac{m}{4\pi} e^{-i\Omega\theta} \int_{-\infty}^{\infty} F(u)\frac{u+\Omega}{u+q} \exp\left[-iu\theta - \mu r_\perp^2 \frac{u+\Omega}{u+q}\right] du, \\ &\theta = t - \frac{z}{c}, \quad q = \Omega\left(1 - \frac{z}{R}\right) + i\frac{z\Omega}{ka_0^2}, \quad \mu = \frac{1}{2}\left(\frac{1}{a_0^2} + \frac{ik}{R}\right), \\ &F(u) = \int_{-\infty}^{\infty} f(t)\exp\{i[ut + \phi(t)]\}dt, \quad E^- \equiv E^-(R) = E^+(-R), \\ &E^\circ = E^+(\infty)m^{-1}. \end{aligned} \tag{4}$$

The three terms in this expression described the focused (E^+), the unfocused (E^0), and the defocused (E^-) parts of the light passing through the zone plate. Let us examine the expression for E^+. At $\Delta\omega/\Omega \ll 1$ (where $\Delta\omega$ is the width of the spectrum of the incident pulse) we can rewrite E^+ on the beam axis ($r_\perp = 0$) in the form

$$E^+ \simeq \frac{m\Omega}{2i} e^{-i\Omega\theta} \int_{-\infty}^{\theta} e^{-iq(\theta'-\theta) + i\phi(\theta')} f(\theta')\, d\theta' \tag{5}$$

It is seen from (5) that if

$$\Delta\omega \gtrsim \Omega \frac{R}{ka_0^2} \tag{6}$$

then the focusing by a zone plate differs appreciably from focusing by the corresponding lens.

We consider first a solitary incident pulse of initial duration τ_0 without phase modulation ($\phi(t) \equiv 0$)[1]. From Eq. (5) under condition (6) it follows that in the region of the focus ($z \simeq R$) the pulse (even if originally symmetrical) becomes asymmetrical in the course of time. The corresponding light train has a steeper leading front and a less sloping trailing edge. Under the stronger condition

$$\tau_0 \ll \frac{a_0^2}{cR} \tag{7}$$

the light train is strongly asymmetrical in the vicinity of the focus, namely, the spatial scale of the leading front amounts to $c\tau_0$, while the corresponding value for the trailing edge is a_0^2/R. Therefore the track of two-photon luminescence of two opposing pulses, one unfocused and one focused by the zone plate, which overlap in the focal region, will have in this region a relief that has the same features as the indicated light train (there will be only one "step" with scale $c\tau_0$ in the interval a_0^2/R)[2]. As seen from (5), an incident pulse can be regarded as solitary if the time interval to the neighboring pulse greatly exceeds a_0^2/cR. On the other hand, if the intervals between the neighboring pulses are smaller than or of the order of a_0^2/cR, then it can be readily verified with the aid of (5) that the character of the relief in the considered track will be qualitatively altered in comparison with the case of a solitary pulse (there will be several "steps" of scale $c\tau_0$ in a single interval a_0^2/R). Let us consider further the case when the incident radiation is a stationary random process with envelope $f(t)$ and phase $\phi(t)$ and with a characteristic correlation time τ_k for the envelope fluctuations satisfying the condition $\tau_k \ll a_0^2/cR$. In this case a typical realization of the random process contains in the interval a_0^2/cR many (several) intensity peaks (the duration of each peak is on the order of τ_k), and therefore the given peaks are not solitary. We consider the track of two-photon luninescence of opposing beams, one unfocused and the other focused by a zone plate, at the same ratio of their average intensities (over the time a_0^2/cR); this ensures the most pronounced relief in this track in the case of a solitary pulse of duration $\tau_0 = \tau_k$. As seen from (5), the values of the envelope fluctuations in the focal region are practically independent of the corresponding values of $f(\theta)$, just as a stationary process and its integral are mutually independent. Therefore the track in question will not have a noticeable relief in comparison of the relief obtained with a solitary pulse.

Thus, the use of a zone plate makes it possible to estimate the duration τ_0 of a solitary pulse from the shape of the relief of the considered track of two-photon luminescence, and to distinguish the case of a solitary pulse in an interval $a_0^2/cR \gg \tau_0$ from the case of two or more pulses in the same interval, and from the case of stationary random incident radiation.

We present also relations that determine, for a single pulse without phase modulation, under the condition (7), the maximum attainable energy density I_f in the focal region, and the value d_f of the focal-spot diameter:

$$I_f \sim \frac{m^2}{8(1+m)^2} (\Omega\tau_0)^2 I_0, \qquad d_f \sim \frac{1}{2} \sqrt{\frac{R}{c\tau_0}}\, \lambda. \tag{8}$$

Here I_0 is the initial intensity on the beam axis. As seen from (8), focusing by a zone plate under the condition (7) is much less effective than focusing by a corresponding lens.

Let us consider a numerical example. We assume $d_0 = 3$ and $R = 30$ cm. Then, according to

(6), the effects in questions come into play at an incident-pulse duration $\tau_0 \lesssim 3\times10^{-12}$ sec.

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1) Here and throughout we have in mind the case of greatest practical interest, when the width of the spectrum of the incident radiation is governed by the time variation of the envelope f(t). This is easy to establish, e.g., by comparing the width of the incident-radiation spectrum with the longitudinal dimension of the relief in the two-photon luminescence track of two identical opposing beams (not focused by a zone plate).

2) To be able to conveniently increase the dimensions of the focal region itself, one can use beams that have been previously defocused (focused) with an ordinary lens and then focused (defocused) with a zone plate.

POSSIBILITY OF DYNAMIC DEFORMATION OF SPHERICAL NUCLEI

V. G. Zelevinskii
Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences
Submitted 18 December 1973
ZhETF Pis. Red. 19, No. 3, 179 — 181 (5 February 1974)

It is demonstrated with a simple model that as the angular momentum of the nucleus is increased, a dynamic instability of the spherical state can set in, and is due to the competition between the pairing and the forces of quadrupole-deformation.

In connection with the observation of regular quasirotational bands in spherical even-even nuclei [1], the question of the change of the internal structure of the nucleus with increasing angular momentum I becomes timely. To attain $I \neq 0$ it is necessary to break the spherical pairs and this can lead to a "phase transition" even if the static deformation in the ground state (I = 0) is energywise not favored. This may be, in particular, the cause of the singularities in the spectra of ^{186}Hg [2] and $^{100,102}Pd$ ("V-event" [3]).

We consider a simple model [4] in which N outer nucleus fill a degenerate shell that includes $2\Omega \ll 1$ states and interact via pairing and quadrupole forces

$$H = H_P + H_Q = -\frac{G}{2} A^+A - \frac{\kappa}{2} \sum_\mu Q_\mu^+ Q_\mu. \tag{1}$$

Here $A = \sum_\nu a_\nu a_{\tilde{\nu}}$ is the Cooper-pair operator, Q_μ is the total quadrupole moment, G and κ are coupling constants. The pairing H_P is diagonalized [5] by changing over to the pseudospin S

$$A = 2(S_x - iS_y), \qquad A^+ = 2(S_x + iS_y), \qquad N = \Omega + 2S_z \ ,$$
$$H_P = \text{const} - 2GS(S+1), \tag{2}$$

where S varies from $\Omega/2$ (seniority $v = \Omega - 2S = 0$) to $N - \Omega/2$, when $v = v_{max} = N$ or $(2\Omega - N)$. The minimum of H_P corresponds to $v = 0$, and the first excited state has $v = 0$ and is separated by a gap $2\Delta = 2G\Omega$. At the same time, neglecting the higher multipoles, H_Q yields a system of rotational bands [4]

$$H_Q \approx -\frac{\kappa}{2} q^2 C + \frac{\kappa}{2} q^2 b\, \mathbf{I}^2 \ , \tag{3}$$

where q is the reduced matrix element of the single-particle quadrupole moment, $b = (6/5)\ [\Omega(2\Omega - 1)(2\Omega + 1)]^{-1}$, and C is the Casmir operator of the SU(3) group. In the band including levels with even angular momenta from 0 to I we have $C = (4/3)b\bar{I}(\bar{I} + 3)$ [6].

In the absence of pairing, static deformation is favored. In the ground band, C is maximal and the limiting angular momentum is equal to

$$\bar{I} = \bar{I}_{max}(N) = \Omega N\left(1 - \frac{N}{2\Omega}\right). \tag{4}$$