$$(\eta - \xi + r)^2 - 4r\eta = 0, \qquad (6)$$

and the equation of parabola II is

$$(\eta - q)^2 - 2p\xi(\eta + q) + p^2\xi^2 = 0. \qquad (7)$$

Parabolas I and II lie entirely in the upper right quadrant. Parabola I is tangent to the axes η and ξ at the points ($\eta = r$, $\xi = 0$) and ($\eta = 0$, $\xi = r$), respectively, while parabola II is tangent to the same axes at the points ($\eta = q$, $\xi = 0$) and ($\eta = 0$, $\xi = q/p$). The relative placement of these parabols in the figure takes into account the inequalities (5). The dynamic NDC can exist only at those points of the (ξ, η) plane lying outside parabola I, which is the region shaded horizontally. The stability at all values of the wave number K is ensured only for points inside the region bounded by parabola II and by the ξ-axis segment to the right of the point $\xi = q/p$ — the vertically-shaded region. It is seen from the figure that these two regions always overlap. Consequently, there is always a region in the (ξ, η) plane where the possibility of dynamic DNC combines with stability of the semiconductor against arbitrary fluctuations. This section is doubly hatched in the figure. We can conclude from definitions (3) and (4) and from the figure that the doubly-hatched region is broader, meaning that experimental observation of the effect is easier if the parameter q is not too small in comparison with the parameter r. This can be attained in practice by decreasing to a minimum the ejection times τ_{g1} and τ_{g2} by using suitable additional illumination. In addition, to obtain the effect it is necessary to have traps that lead to a sublinear static current-voltage characteristic ($\sigma_d(0) < en_0\mu$).

We have also calculated the impedance $Z(\omega)$ of a semiconductor with two types of traps, but with antibarrier contacts on the end. It turned out that even with such contacts it is possible to obtain simultaneously both $\mathrm{Re}Z(\omega) < 0$ and stability against fluctuations. We considered also another system, namely a semiconductor with inertially heated electrons and with impurities under conditions close to breakdown. For this system we were also able to confirm the feasibility of dynamic NDC in conjunction with fluctuation stability. We emphasize that this effect vanishes if traps of one type are removed in our first example, and if in our second example we remove the impurity or make the heating instantaneous. Thus, we are dealing with an effect typical of systems in which the relaxation to equilibrium is described not by one but by at least two different times. In our examples these were either different recombination times at the two types of traps, or an energy relaxation time different from the breakdown time.

The feasibility of dynamic NDC in homogeneous semiconductors was investigated in [1]. However, the dynamic NDC mechanism which we examined in detail for monopolar semiconductors with traps of two types was not discussed in [1]. In addition, and this is the crux of the matter, in our article we indicate, apparently for the first time, that dynamic NDC is possible in semiconductors that are stable against fluctuations.

The author is deeply grateful to M. S. Kagan and S. G. Kalashnikov for constant interest in the work and discussions, and to V. L. Bonch-Bruevich for discussions.

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GROWTH OF TOTAL CROSS SECTIONS AND ELASTIC SCATTERING

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Submitted 24 December 1973
ZhETF Pis. Red. 19, No. 3, 185 – 188 (5 February 1974)

A simple expression is proposed for the amplitude of elastic proton-proton scattering; this expression corresponds to the Froissart growth of the total cross section and describes satisfactorily all the differential cross section data obtained with the CERN storage rings.

Measurements at CERN have shown [1] that the total cross section of proton-proton scattering increases rapidly with energy. At the same time, the slope parameter of the diffraction peak and the ratio of the elastic cross section to the total cross section vary slowly with energy [2, 3, 1]. We shall show that this behavior, at the attained energies, is fully compatible with the Froissart growth of the total cross section.

As $s_0 < s < 3000$ GeV^2 the total cross section is adequately described by the formula

$$\sigma = \sigma_0 + \Delta , \tag{1}$$

$$\Delta = \sigma_1 \ln^2 (s/s_0), \tag{2}$$

where $\sigma_0 = 38.4$ mb, $s_0 = 122$ GeV^2 and $\sigma_1 = 0.49$ mb. The parameters s_0 and σ_1 can vary somewhat at a fixed value of Δ in the measured region. We use the parameters of [4], which correspond to cosmic-ray data. This behavior is compatible with the maximum possible (Froissart) growth. It is of interest to assume that the term Δ corresponds to saturation of the Froissart limit, and σ_0 takes effective account of the pre-asymptotic terms. But then we find that the slope parameter of the diffraction peak $b(s, t) = (d/dt)\ln(d\sigma_{el}/dt)$ and the ratio of the total elastic cross section to the total cross section, $r(s) = \sigma_{el}/\sigma$, should behave in a definite manner as $s \to \infty$:

$$b(s \to \infty, 0) = (16\pi\rho)^{-1}\Delta, \tag{3}$$

$$r(s \to \infty) = \rho . \tag{4}$$

Here ρ is a constant, which can be reasonably assumed equal to 1 (minimal inelastic contribution) or to 1/2 (maximal inelastic contribution). Should the relations (3) and (4) hold at the attainable energies? How can the transition to the Froissart limit be realized for b and r if it is realized for σ in accordance with formula (1)? To answer these questions, we consider the elastic-scattering model corresponding to (1). Since the elastic cross section decreases rapidly with increasing -t, it suffices, even for the analysis of σ_{el}, to confine ourselves to $-t < 0.3$ (GeV/c)^2. We can then neglect the real part of the elastic amplitude. We consider the amplitude

$$T(s,t) = i\,[\sigma_0 \exp(b_0 t/2) + \Delta 2(-4b_1 t)^{-1/2} J_1((-4b_1 t)^{1/2})] , \tag{5}$$

where J_1 is a Bessel function. From the conditions (3) and (4) we have

$$b_1(s \to \infty, 0) = (16\pi\rho)^{-1}\Delta. \tag{6}$$

To choose b_0, we note that the data at $s < s_0$ are well described by the first term of (5), so that we can choose for b_0 the Serpukhov value [2] of the slope parameter. At $-t < 0.14$ (GeV/c)^2 we have

$$b_0(s, t) = [6{,}8 + 2 \cdot 0{,}47 \ln(s/1\ \mathrm{GeV}^2)]\ (\mathrm{GeV/c})^{-2} \tag{7}$$

Taking into account in b_1 the pre-asymptotic terms with the aid of the constant β, we have

$$b_1(s, t) = \beta + (16\pi\rho)^{-1}\Delta . \tag{8}$$

The inclusion of β is an essential feature of the model considered here, in which β is the only free parameter. Choosing

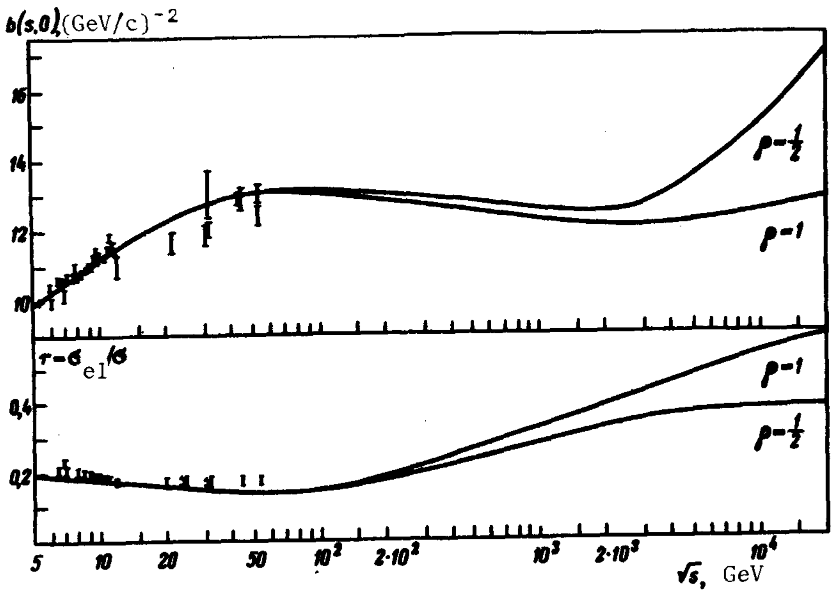
$$\beta = 2{,}5\ (\mathrm{GeV/c})^{-2}, \tag{9}$$

we arrive at the following consequences:

1. The growth of the diffraction-peak slope parameter b(s, 0) slows down with increasing energy, and the parameter remains practically constant at a level 12 - 13 (GeV/c)$^{-2}$ in the range of s from 10^3 to 10^7 GeV^2 (see the figure).

2. With increasing -t and at fixed s, the parameter b decreases. Thus, at $s = 2500$ GeV^2 its values are 13.0, 12.0, and 10.5 (GeV/c)$^{-2}$ at $-t = 0$, 0.1, and 0.2 (GeV/c)^2, respectively.

3. The ratio of the elastic cross section to the total cross section, r, stays at the level 0.1 - 0.2 in the interval of s from 100 to 10^5 GeV^2 (see the figure).



Diffraction-peak slope parameter at $t = 0$, and the ratio of the elastic cross section to the total cross section for pp scattering in the case of the Froissart mechanism. ρ is the asymptotic value of r. The experimental points are from [1 - 3].

Thus, in accord with the available data, the parameters b and r approach their Froissart limits very slowly. As noted in [6], this is due to the smallness of the numerical coefficient in the asymptotic formula (3). We see that the hypothesis of the Froissart growth of the total proton-proton cross sections agrees with all the available data, and a check of the universality [6] of this behavior in other reactions would be of great interest.

We note in conclusion that the amplitude (5) is polynomially bounded at fixed $t > 0$, satisfies unitarity in the direct channel, and has in the j-plane a cut $[(j - 1)^2 - (\sigma_1/4\pi\rho)t)^{-3/2}$. At large -t, it gives rise to cross-section minima and secondary maxima, which shift towards smaller -t with increasing energy. Thus, at $s = 900\ \text{GeV}^2$ and $\rho = 1/2$ the first minimum, the second maximum, and the second minimum are located at $-t = 1.3$, 2.4, and 4.5 $(\text{GeV/c})^3$, respectively. At $s = 2500\ \text{GeV}^2$ they shift to 1.2, 2.2, and 4.1 $(\text{GeV/c})^2$, respectively. Multiplying the second term of (2) by $\exp(ct/2)$, where $c = 3.3\ (\text{GeV/c})^{-2}$, we obtain a satisfactory description of the differential proton-proton scattering cross section in the range of measurements in the CERN storage rings, where the cross sections change by seven orders of magnitude.

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