Nonlinear stage of parametric wave excitation in a plasma

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The spectra of Langmuir turbulence excited by a high-frequency electric field are investigated. It is shown that at large external-field amplitudes the distribution of the oscillations is essentially nonstationary. This leads to an oscillatory character of the energy absorption in the plasma.

1. When a periodic electric field $E=E_0\cos\omega_0 t(\omega_0 \gtrsim \omega_p)$ of sufficient amplitude is applied to a homogeneous isotropic plasma, plasma waves are parametrically excited.[1-5] If $T_i \lesssim T_e$, the strongest parametric instability leads to a buildup of waves near the k-space surface described by the equation

$$\omega_0 = \omega_k + s|k|. \tag{1}$$

Here $\omega_k=\omega_p(1+\frac{3}{2}k^2r_D^2)$ is the law of dispersion of the Langmuir waves, and $s=\sqrt{T_e/M}$ is the velocity of the ion sound. At not too large field amplitudes

$$\left(\frac{E_0^2}{8\pi n T_e} < \sqrt{\frac{m}{M}}\, k^* r_D \left(\frac{\gamma_s}{s k^*}\right)^2,\right.$$

where γ_s is the damping of the ion sound, $k_{dif} \ll k^* \ll 1/r_D$ is the characteristic wave number of the excited waves, $k_{dif}=\sqrt{(m/M)}(1/r_D)$, the principal role in the nonlinear limitation of the instability is played by the induced scattering of the Langmuir waves by the plasma ions. This plasma can lead to a wave-energy flux in the region of small wave numbers. At not too large increments of the parametric instability $\Gamma_k < \gamma_L(k^*/k_{dif})$, the Langmuir-wave energy dissipation is ensured by their linear (collisional) damping γ_L, and when $\Gamma_k \gg \gamma_L(k^*/k_{dif})$ the energy dissipation occurs in regions of small wave numbers and is ensured by a nonlinear dissipation mechanism, namely the collapse of the Langmuir waves.[6] In this case, a region of inertial transfer of the wave energy exists at $k > k^*$.

The angular anisotropy of the instability increment Γ_k causes the spectrum of the Langmuir waves to become quasi-one-dimensional, and take the form of symmetrical "jets" elongated in the direction of E_0 (this was proved in [4] for problems with a characteristic scale $\Delta k \gg k_{dif}$). This enables us to confine ourselves to the consideration of the one-dimensional problem. In the one-dimensional symmetrical case the kinetic equation for the waves is

$$\frac{\partial n_k}{\partial t} = n_k\{\Gamma_k + \int_0^\infty T(k-k')n_{k'}dk' - \gamma_L\} + \gamma_L n_0. \tag{2}$$

Here $\Gamma_k=[\omega_p E_0^2/8\pi n T]\phi(\xi)$ is the instability increment, $T(\xi)=[\omega_p^2/2n_0T]\phi(\xi/2)$ is the matrix element of the induced scattering, and $\phi(\xi)=-\phi(-\xi)$ is a dimensionless structure function such that $T(k-k')$ has sharp extrema at $k-k'=\pm k_{dif}$. At $T_i/T_e \ll 1$ we have

$$\phi(\xi) = \mathrm{Im}\frac{1}{4\frac{T_i}{T_e}\xi^2 - 1 + \sqrt{\frac{2\pi m T_i}{M T_e}}\,\xi}$$

where n_0 is the amplitude of the thermal noise.

2. Equation (2) was simulated with a computer, using 100 points in the interval from $k=k^*$ to $k=0$; in the region of the first 10 points, strong linear damping was turned on and guaranteed absorption of the energy condensed in the region of small wave numbers. The numerical experiment has shown that in all cases the one-dimensional spectrum consisted of a chain of narrow ($\Delta k \ll k_{dif}$) peaks located at distances k_{dif} from one another.[1)] The peak width decreased with decreasing T_i/T_e. It is possible here to separate two cases. At not too large instability increments $\Gamma/\gamma_L < k^*/k_{dif}$, a stationary state is established in the form of a sequence of peaks that decreases linearly to zero (Fig. 1). This result agrees with the well known results of Oberman, Valeo, and Perkins.[2,5] The time required to establish the stationary state is inversely proportional to the noise level n_0 and is of the order of $(1/\tau)\sim\gamma_L(n_c/n_0)$, where $n_c \approx \Gamma/T$ is the characteristic amplitude of the parametric waves. At large excesses above the instability threshold $(\Gamma/\gamma_L > k/k_{dif})$, no stationary state is established, and a relaxation process periodic in time is observed instead; several of the states of this process, following each other in time, are shown in Fig. 2. The energy release in the $k\sim k^*$ zone then takes the form of pulses that propagate subsequently in the region of small k along a chain of peaks, in the form of localized excitations of the chain. The maximum peak amplitude is of the order of $n_c\ln(n_c/n_0\Delta k)$, where Δk is the width of the peak, and the time interval between peaks is of the order of $\sim 1/\gamma_L\ln(n_c/n_0\Delta k)$. The pulse propagation velocity is of the order of $v\sim\Gamma_k k_{dif}$ and depends little on the noise amplitude.

3. The existence of a discrete chain of peaks makes it possible to replace Eq. (2) by a finite-difference equation that has the following form in terms of the dimensionless variables f_n (f_n is the amplitude of the nth peak):

$$\frac{\partial f_n}{\partial t} = f_n(f_{n+1} - f_{n-1} - \gamma_n + \Gamma_n) + C_n. \tag{3}$$

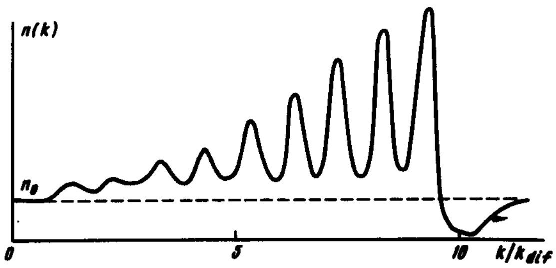

FIG. 1. Distribution of n_k at large times, $\gamma_L t=100$. The excess over threshold is $\Gamma_k/\gamma_L=4.37$.

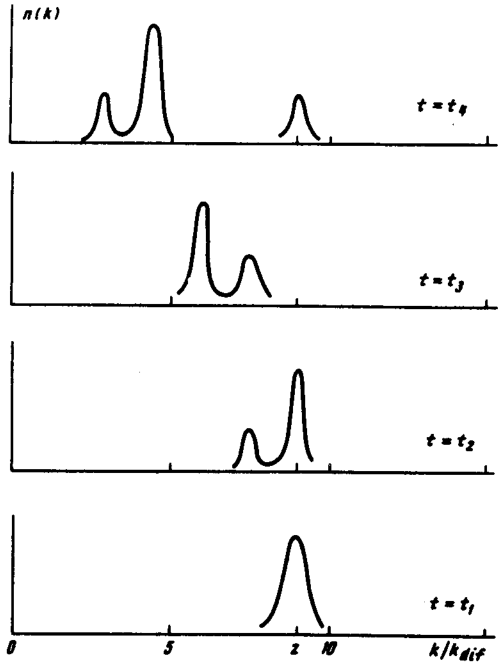

FIG. 2. Distribution of n_k at infinite excess above threshold ($\gamma_L = 0$) for successive instants of time ($t = t_1, t_2, t_3, t_4$) in arbitrary units. The point $k = z$ corresponds to the maximum of the increment.

In the inertial region, neglecting the linear damping and the noise, we have

$$\frac{\partial f_n}{\partial t} = f_n(f_{n+1} - f_{n-1}). \qquad (4)$$

Equation (4) has an exact solution $f_n(t) = f(t - n/s - \tau_0)$, where

$$f(\xi) = f_0\left(1 + \frac{a}{1 - b + b\,\mathrm{ch}\,\gamma\xi}\right). \qquad (5)$$

Here f_0, a, and τ_0 are arbitrary parameters, while s, b, and γ are functions of a and f_0; when $a \gg 1$ we have

$$\gamma = 2f_0 a; \qquad b^2 = \frac{1}{a}; \qquad \frac{\gamma}{s} = \ln a.$$

The solution (5) is a soliton propagating in k-space along a chain of peaks. The nonstationary process observed in the numerical experiment at $\Gamma/\gamma_L > k^*/k_{dif}$ can be visualized as a process of successive "detachment" of the solitons from the instability zone k^*, their propagation in the inertial region, and their "annihilation" in the region of small wave numbers. The soliton propagation velocity is $v \sim 2k_{dif}\Gamma/\ln\ln(n_c/n_0\Delta k)$. A similar qualitative character is possessed by the initial stage of the process of establishment of the stationary state at $\Gamma/\gamma_L < k^*/k_{dif}$; in this case, however, the solitons are damped and stop before reaching the region of small k.

We note that the nonstationary character of the spectrum of the Langmuir waves should lead, in experiments on parametric excitation, to oscillations of the absorption of the high-frequency field energy in the plasma. The time-averaged energy flux into the plasma coincides then with the value obtained in [4] within the framework of the diffusion approximation, and the results of the present paper can be regarded as an investigation of the fine structure of spectra of the "jet" type.

It can be shown that the difference system (4) is fully integrable, and that the soliton (5) is a stable formation.

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1)The possible existence of spectra of this type was first suggested in a paper by Kingsep and Rudakov.[7]

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