Magnetic freeze-out in semiconducting BiSb alloys

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Using BiSb alloys as an example, it is shown that magnetic freeze-out in semiconductors with strongly anisotropic equal-energy surfaces acquires qualitative singularities.

The study of magnetic freeze-out (MF) in semiconductors has been the subject of many investigations (see, e.g., the review[1]). Insofar as we know, however, all these investigations pertain to materials in which the energy spectrum of the electrons is close to isotropic (compounds of the III-V type[1]). In the present paper we prove on the basis of a very simple model and confirm experimentally that the MF phenomenon in semiconductors with strongly anisotropic equal-energy surfaces acquires qualitative singularities.

We consider for the sake of simplicity a semiconductor with a quadratic dispersion law and an equal-energy surface in the form of a strongly elongated ellipsoid of revolution, such that $m_\perp = m_x = m_y$ and $m_z = m_\parallel \gg m_\perp$. The Hamiltonian of the electron in the presence of a strong magnetic field and a Coulomb center is of the form

$$\hat{H} = \frac{1}{2m_\perp}\left(\left(\hat{p}_x - \frac{e}{c}A_x\right)^2 + \left(\hat{p}_y - \frac{e}{c}A_y\right)^2\right) + \frac{1}{2m_\parallel}\hat{p}_\parallel^2 - \frac{e^2}{\epsilon_0\sqrt{\rho^2+z^2}} \quad (1)$$

where the gauge most convenient for our purposes is $A = (1/2)\mathbf{H}\times\mathbf{r}$, and $\hbar = 1$. In analogy with the procedure developed in [2], we seek a ground-state wave function in the form $\psi(\rho, z) = \phi_{00}(\rho)u(z)$, where

$$\phi_{00}(\rho) = \frac{\sqrt{2}}{a_H}\exp\left(-\frac{\rho^2}{2a_H^2}\right), \quad a_H^2 = \frac{2c}{eH}. \quad (2)$$

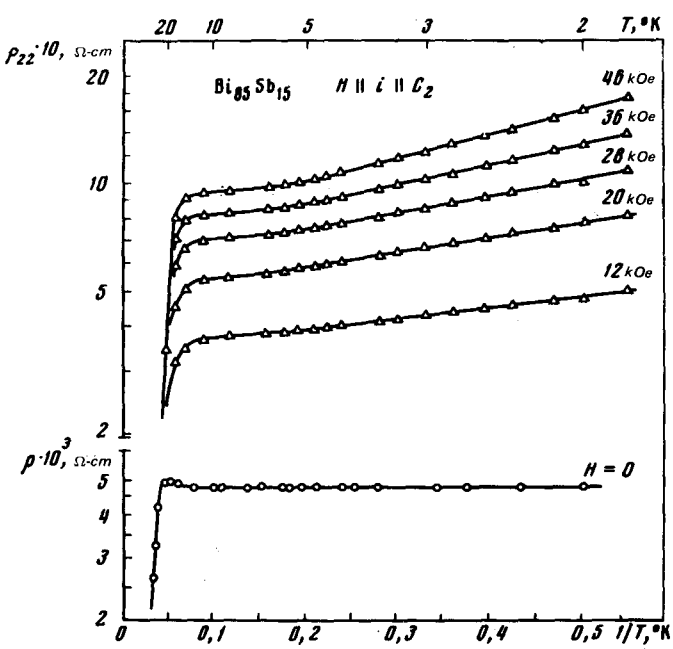

FIG. 1. Temperature dependences of the longitudinal magnetoresistance $\rho_{22}(T)$ for the sample $Bi_{85}Sb_{15}$.

Averaging (1) over (2), we obtain a one-dimensional effective potential

$$\bar{V}(z) = -\frac{2e^2}{a_H^2\epsilon_0}\int_0^\infty \frac{\rho\,d\rho}{\sqrt{\rho^2+z^2}}\exp\left(-\frac{\rho^2}{a_H^2}\right) = \begin{cases} -\dfrac{\sqrt{\pi}}{\epsilon_0}\dfrac{e^2}{a_H^2} + \dfrac{2e^2}{\epsilon_0}\dfrac{|z|}{a_H^2}, & |z| \ll a_H \\ -\dfrac{e^2}{\epsilon_0}\dfrac{1}{|z|}, & |z| \gg a_H \end{cases}. \quad (3)$$

In fields H determined by the condition $a_H \leqslant a_\parallel \equiv \epsilon_0/m_\parallel e^2$, we are dealing with the situation adequately described by the formula of [2]. The binding energy is given by $E = (m_\parallel e^4/\epsilon_0^2)f(a_\parallel/a_H)$, where f is a function that increases slowly with the field.

In the opposite region of the fields H, i.e., at $a_H \gg a_\parallel$, the electron is localized mainly near the minimum of $\bar{V}(z)$, and the binding energy is

$$E = \frac{\sqrt{\pi}}{\epsilon_0}\frac{e^2}{a_H} = \frac{e^2}{\epsilon_0}\left(\frac{\pi eH}{2c}\right)^{1/2}. \quad (4)$$

The amplitude of the "zero-point" oscillations is of the order of

$$(a_\parallel a_H^2)^{1/3} \ll a_H. \quad (5)$$

We note that the range of fields in which formula (4) is valid is completely determined by the inequalities $a_\parallel \ll a_H \ll a_\perp \equiv \epsilon_0/m_\perp e^2$. If this condition is violated in the region of weak fields, the binding energy ceases to depend significantly on H, and is determined by a quantity on the order of $m_\perp e^4/\epsilon_0^2$.

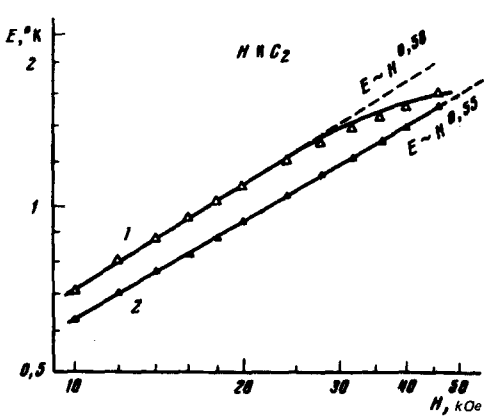

FIG. 2. Binding energy $E(H)$ vs the magnetic field for the samples $Bi_{83}Sb_{17}$ (curve 1) and $Bi_{85}Sb_{15}$ (curve 2).

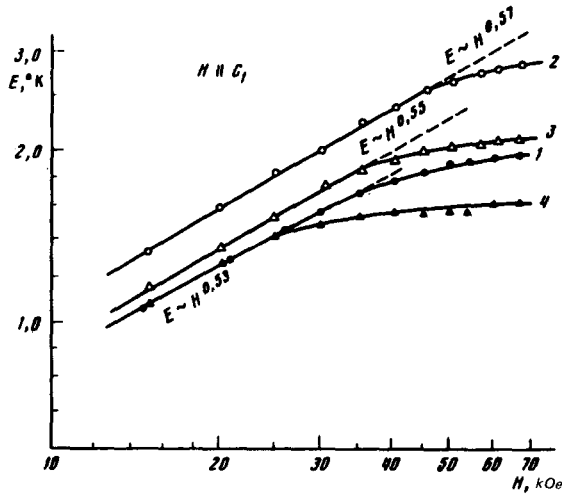

FIG. 3. Binding energy $E(H)$ vs the magnetic field for the samples $Bi_{95.8}Sb_{4.2}$ and $Bi_{95.4}Sb_{4.6}$, measured at different pressure P: 1) $Bi_{95.8}Sb_{4.2}$, $P = 14.8$ kbar; 2) $Bi_{95.8}Sb_{4.2}$, $P = 14.7$ kbar; 3) $Bi_{95.4}Sb_{4.6}$, $P = 14.1$ kbar; 4) $Bi_{95.4}Sb_{4.6}$, $P = 13.2$ kbar.

We call attention to the fact that Eq. (4) contains no characteristics of the crystal, other than the dielectric constant ϵ_0 and $E \sim H^{1/2}$, i.e., the dependence of E on H in a semiconductor having an anisotropic equal-energy surface, similar to that considered here, is much stronger than in a semiconductor with an isotropic equal-energy surface, where $E(H)$ is approximated in a wide range of H by the relation $E \sim \ln^2 H$. [2]

It is of interest to observe the MF in materials with strongly anisotropic equal-energy surfaces. To this end, we have investigated semiconducting single-crystal n-BiSb alloys, in which the electron equal-energy surfaces are three triaxial ellipsoids inclined at an angle ~6° to the basal plane and rotated 120° relative to one another (C_1, C_2, and C_3 are respectively the binary, bisector, and trigonal axes). In these materials, the concentration $n = N_d - N_a$ of the impurity electrons in the conduction band remains constant at $H = 0$ down to the lowest temperatures, since the electron binding energy in the impurity center is very low, $E \leq 10^{-2}$ °K, and furthermore the screening length is $\lambda \approx 10^{-5}$ cm, which is smaller by one order of magnitude than the Bohr radius $a_B \approx 10^{-4}$ cm, i.e., there are no bound states.

Figure 1 shows the temperature dependences of the longitudinal magnetoresistance $\rho_{22}(T)$ of the n-$Bi_{85}Sb_{15}$ sample, measured in the intervals $1.82 \leq T \leq 30$ °K and $0 \leq H \leq 46$ kOe ($n \approx N_d - N_a \approx 7 \times 10^{14}$ cm^{-3} and the forbidden band width is $\epsilon_g \approx 200$ °K). At $H = 0$ and $1.82 \leq T \leq 15$ °K we have $\rho_{22}(T) \approx$ const, since $n(T) \approx$ const (see above) and, in addition, $\mu(T) \approx$ const because the electrons are degenerate at $n \approx 10^{14-15}$ cm^{-3} the Fermi energy is $\epsilon_F \approx 2-3$ meV. [3] The *s-arp* decrease of $\rho_{22}(T)$ at $T \gtrsim 20$ °K is due to the onset of intrinsic conductivity. In strong H, the behavior of $\rho_{22}(T)$ changes qualitatively, namely, at low temperatures $1.82 \lesssim T \lesssim 5-6$ °K the longitudinal magnetoresistance depends on $1/T$ exponentially [$\rho_{22}(T) \sim \exp(E/T)$], while E increases like $\sim H^{1/2}$ (Fig. 2) and is of the order of several degrees. In the intermediate region $6° \lesssim T \lesssim 15-20$ °K the value of ρ_{22} depends little on T, and at $T \gtrsim 20$ °K we have $\rho_{22}(T) \sim \exp(\epsilon_g/2T)$. The measurements of $\rho_{22}(T)$ were made with the currents flowing through the sample corresponding to the ohmic section on the current-voltage characteristic.

We assume that the relation $\rho_{22}(T) \sim \exp(E/T)$ in the low-temperature region is the consequence of MF at the impurity levels, i.e., that $n \sim \exp(-E/T)$ at low temperatures. The estimates that follow confirm this possibility. Indeed, in fields $H \gtrsim 1$ kOe we have $\hbar\omega \approx \epsilon_F \gg kT$ and the electrons are at the lowest Landau level, on which the density of states N_c increases linearly with H and turns out to be $N_c \gtrsim n$, the degeneracy is lifted, and furthermore the characteristic dimension of the electron wave function is now determined by the magnetic length $a_H \lesssim \lambda$, i.e., bound states become possible. Thus, fields $H \geq 1$ kOe alter radically the MF conditions in the semiconducting BiSb alloys and make the MF possible.

At the orientation $\mathbf{H} \parallel C_2$ the magnetic field is directed along the long axis of one of the three electron ellipsoids, and we have [5] $m_1 \approx 3.5 \times 10^{-3} m_0$, $m_2 \approx 8.6 \times 10^{-1} m_0$, $m_3 \approx 4.6 \times 10^{-3} m_0$, and $\epsilon_0 \approx 400$, [4] while the region of H in which (4) is valid is bounded by the values $H_{min} \approx 30$ Oe and $H_{max} \approx 200$ kOe. We can thus estimate E by using the MF model considered at the beginning of the paper, since E is of the order of several degrees and is close to the observed quantity.

An estimate of E for the other two ellipsoids, the long axes of which are not parallel to H, gives a value of the same order of magnitude. It is important to emphasize that the dependence of E on H is preserved in this case. As will be shown in a more detailed paper, the slope of H relative to the axis of elongation of the ellipsoid (with the exception of a narrow angle interval near $\mathbf{H} \perp z$) and the anisotropy of the transverse masses ($m_x \neq m_y$) leads to the appearance in (4) of a numerical factor on the order of unity and independent of H.

Figure 2 shows the dependence of E on H for n-$Bi_{83}Sb_{17}$ samples with $n = N_d = N_a \approx (7-8) \times 10^{14}$ cm^{-3}, obtained from the dependences of ρ_{22} on T at low temperatures and in fields $H \lesssim 46$ kOe. In fields $H \lesssim 30$ kOe we have $E \sim H^{0.58}$, and at $H \gtrsim 30$ kOe a somewhat slower rate of $\partial E/\partial H$ is observed, possibly because the fields $H \gtrsim 30$ kOe turn out to be close to the region of H_{max}.

Figure 3 shows plots of E against H for the alloys $Bi_{95.8}Sb_{4.2}$ and $Bi_{95.4}Sb_{4.6}$, which were transformed from the semimetallic phase into the semiconducting one by application of a hydrostatic pressure P. These results were obtained from the low-temperature sections of the plots of ρ_{11} against T, measured at different P and H. At a given orientation of $\mathbf{H}$, the electron binding energy in the valley with the elongation perpendicular to $\mathbf{H}$ is negligibly small in comparison with the binding energy of the electrons in the two other valleys. Thus, the MF picture remains qualitatively the same at two different orientations of H relative to the crystallographic axes, $\mathbf{H} \parallel C_2$ (Fig. 2) and $\mathbf{H} \parallel C_1$ (Fig. 3).

It must be emphasized that the growth rate $\partial E/\partial H$ (Figs. 2 and 3) is close to (4), at least in some interval of H, and is much larger than the $\partial E/\partial H$ observed for

semiconductors with isotropic equal-energy surface.[11]

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