Oscillations of the phase of the order parameter upon propagation of sound in a superconductor

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It is shown that oscillations of the order-parameter phase set in when longitudinal sound propagates along a thin current-carrying superconducting wire. These oscillations can be observed with the aid of a superconducting magnetometer.

According to numerous experimental data, the superconducting transition temperature T_c depends on the pressure or, equivalently, on the relative change of the volume (see e.g., [1, 2]). It is therefore clear that when longitudinal sound propagates in a superconductor, the gap Δ will oscillate. The scale of alteration of the gap by the sound, at sufficiently low temperatures ($T \ll T_c$), can be easily related with quantities known from experiments on the effect of pressure on superconductivity. To this end we use the relation between Δ and T_c at low temperatures

$$\Delta = \frac{\pi}{\gamma} T_c \tag{1}$$

where $\ln\gamma = C = 0.577$. It is seen from (1) that the correction Δ_1 to the gap is connected with the relative change of volume as follows:

$$\frac{\Delta_1}{\Delta} = -\frac{1}{\kappa T_c}\frac{\partial T_c}{\partial p}\,\mathrm{div}\,\mathbf{u}, \tag{2}$$

where $\kappa = -(1/V)(\partial V/\partial p)_T$ is the compressibility, $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$ is the lattice displacement vector, and $\mathbf{k}$ and ω are the wave vector and the frequency of the sound. The values of κ and $\partial T_c/\partial p$ are well known from pressure experiments. We note that formula (2) is valid in the absence of delay effects, which generally speaking take place when sound of finite frequency propagates. We, however, are interested in relatively low frequencies ($\sim 10^5$ sec^{-1}), at which the delay can be neglected.

We consider a superconducting wire of radius R such that

$$l \ll R \ll \delta_L$$

Here l is the electron mean free path and δ_L is the London depth of penetration of the field. The inequality $l \ll R$ enables us, in particular, to disregard the character of electron reflection from the sample walls, while the condition $R \ll \delta_L$ ensures that the problem is one-dimensional. It follows from the latter that the electroneutrality condition

$$\mathrm{div}\,\mathbf{j} = 0 \tag{3}$$

is equivalent to constancy of the current density $\mathbf{j}$ along the wire.

Let longitudinal sound propagate along the wire and

let current flow in it. The connection between the current and the field in a dirty superconductor is given by

$$\mathbf{j} = -\frac{c}{4\pi}\delta_L^{-2}\mathbf{Q} + \sigma\mathbf{E} \tag{4}$$

Here $\mathbf{Q} = \mathbf{A} - (c/2e)\nabla\chi$ is a gauge-invariant quantity, in which $\mathbf{A}$ is the vector potential and χ is the phase of the gap, $\delta_L = (c/2\pi)[\Delta\sigma\tanh(\Delta/2T)]^{-1/2}$ is the London depth of penetration, σ is the conductivity of the metal in the normal state, and $\mathbf{E}$ is the electric field intensity. If the current frequency is $\omega_0 \ll \Delta$, then the second term of (4) can be neglected.

We consider first the case of direct current. Since the sound causes oscillations of Δ, and hence of δ_L (the contribution of the oscillations of σ to the current can be neglected for most superconductors), it is seen from (3) and (4) that the phase χ of the gap should oscillate at the sound frequency ω. Consequently, a phase difference

$$\delta\chi(t) = \frac{16e\pi}{\hbar c^2}\,\frac{j_0\delta_{LO}^2}{\kappa T_c}\,\frac{\partial T_c}{\partial p}\,u_0 \sin\omega t. \tag{5}$$

appears on the ends of a wire segment of length equal to an odd number of sound half-waves. Here δ_{LO} is the London depth in the absence of sound. If an alternating current of frequency ω_0 flows through the wire instead of the direct current j_0, then χ will oscillate at the combination frequencies $|\omega \pm \omega_0|$, among which there will be the zeroth harmonic at $\omega_0 = \omega$. Let us estimate the possible scale of the effect. Let $\omega \sim 10^5$ sec^{-1}, $j_0 \sim 10^6$ A/cm^2, $\delta_L \sim 300$ Å, and let the flux density of the sound energy be $W \sim 10$ W/cm^2. Substituting these values in (5) we obtain

$$|\delta\chi| \sim 10 \text{ deg}.$$

If two points of the sample with different phases are connected by a thin ($R \ll \delta_L$) superconducting wire, then an alternating current will flow in the formed closed loop, and the magnetic field of this current can be measured with a superconducting magnetometer. We note that the magnitude of the described effect at medium temperatures $T \lesssim T_c$ may be of the same order as that of the acoustoelectric effect predicted by Gal'perin, Gurevich, and Kozub.[3] However, the acoustoelectric effect is proportional to the concentration of the normal carriers, decreasing like $\exp(-\Delta/T)$ as $T \to 0$ and increasing like $(1 - T/T_c)^{-1}$ near T_c. In our case the effect is proportional to the current density, and it is therefore convenient to use currents on the order of the critical value. Consequently, the effect increases with decreasing temperature; thus, near T_c it behaves like $(1 - T/T_c)^{1/2}$, and at lower temperatures ($T \ll T_c$) it depends little on T.

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[1]N. B. Brandt and N. I. Ginzburg, Zh. Eksp. Teor. Fiz. 50, 1260 (1966) [Sov. Phys.-JETP 23, 838 (1966)].
[2]R. J. Boughton, J. L. Olsen, and C. Palmy, Progress in low Temperature Physics, 6, 163 (1970).
[3]Yu. M. Gal'perin, V. L. Gurevich, and V. I. Kozub, Zh. Eksp. Teor. Fiz. 65, 1045 (1973) [Sov. Phys.-JETP 38, No. 3 (1974)].