Spontaneous magnetization of electronic thermal conductivity in a laser plasma

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(Submitted January 28, 1974)
ZhETF Pis. Red. **19**, 288–291 (March 5, 1974)

We have investigated theoretically the magnetization of the electronic thermal conductivity in a laser plasma by the spontaneous magnetic fields resulting from thermoelectric effects in an inhomogeneous plasma. We derive a system of equations describing the evolution of the magnetic fields and of the electron temperatures. The increments of the magnetothermal instability are obtained. The role of magnetothermal effects in experiments with laser plasma is discussed.

1. Spontaneous magnetic fields can arise in a plasma produced by laser radiation. These fields were observed experimentally in [1,2]. Certain possible mechanisms leading to these fields have been considered in the literature. In particular, the authors of [2] have assumed that the magnetic fields observed by them are due to the action of the thermoelectric power in an inhomogeneous plasma. In the present paper we consider this mechanism in detail and show that the magnetization of the electrons by the spontaneous magnetic fields can be essential for thermonuclear research with a laser plasma.

2. To be specific, we confine ourselves to the following characteristic plasma parameters: density $n \sim 10^{22}$–10^{23} cm^{-3}, electron temperature $T_e \sim 5$ keV, dimension $a \sim 10^{-2}$ cm, and expansion time $t_{\rm exp} \sim 10^{-9}$ sec. For such a plasma, the magnetic Reynolds number is $\mathrm{Re}_m = 4\pi\sigma a^2/c^2 t_{\rm exp} \sim 10^4$, so that the friction between the electrons and ions can be neglected. At the indicated parameters the mean free path l is much less than a, and the viscosity of the electrons can be neglected. We write the equation of motion of the electrons in the form

$$n m \dot{\mathbf{v}} = -en(\mathbf{E} + \mathbf{E}_{\rm ext}), \qquad (1)$$

where $\mathbf{E}$ is the electric field and $en\mathbf{E}_{\rm ext}$ stands for the sum of all the remaining forces. Substituting (1) in the quasistationary Maxwell equations and recognizing that $a \gg c/\omega_p$ ($\sim 10^{-6}$ cm at $n = 10^{23}$ cm^{-3}), we obtain for the magnetic field generated in the plasma by the extraneous forces

$$\frac{\partial \mathbf{B}}{\partial t} = c \operatorname{rot} \mathbf{E}_{ext} \quad (2)$$

In sufficiently weak magnetic fields we have

$$\mathbf{E}_{ext} = -\frac{1}{e}\frac{\nabla p}{n} + 0{,}7\frac{1}{e}\nabla T_e \quad (3)$$

It follows from (2) and (3) that

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{c}{e}[\nabla T_e \times \nabla \ln n]. \quad (4)$$

Thus, in the case of noncollinear gradients of n and T, the magnetic field in the plasma grows spontaneously. If the angle between the gradients is of the order of unity, then during the time of expansion the magnetic field grows, according to (4), to a value $\sim (c/e)(T/a^2)t$, which would amount to 10 megagauss for the indicated parameter. A magnetic field of this magnitude could affect substantially the hydrodynamics of the expansion ($H^2/8\pi \sim nT_e$).

3. The magnetic fields can, however, magnetize the electrons much sooner (for the considered example we have $\Omega_e\tau_e \sim 1$ at $H \sim 4\times10^4$–4×10^5 G). It is then necessary to express the thermal force in (3) at finite $\Omega_e\tau_e$; in addition, the magnetization of the thermal conductivity makes it necessary to solve simultaneously the equations of heat conduction and the equation for generation of magnetic fields. It is easy to estimate from (4) that $\Omega_e\tau_e$ grows to a value on the order of unity within a time $\sim ma^2/T_e\tau_e$. This is the characteristic time of the electronic thermal conductivity, and from the very meaning of laser heating it should be much shorter than the expansion time. Therefore during the times of interest to us the plasma can be regarded as immobile, and its density can be regarded as a specified function of the coordinates. The expression for the noted inequality is $l/a \gg \sqrt{m/M}$. This enables us also to neglect the energy exchange between the electrons and ions. In addition, at $c/\omega_p \ll l$, the current velocity of the electrons is less than the value $T_e\tau_e/ma^2$ characteristic of the considered processes. This makes it possible to neglect the terms containing the electron velocity and to use the following system of equations, which describes the magnetothermal phenomena in a laser plasma:

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{c}{e}[\nabla T_e \times \nabla \ln n] + \frac{c}{e}\operatorname{rot}\frac{\mathbf{R}_T}{n}, \quad (5)$$

$$\frac{3}{2}n\frac{\partial T}{\partial t} = -\operatorname{div}\mathbf{q} + S. \quad (6)$$

Here $R_T = -\beta_\parallel^{uT}\nabla_\parallel T_e - \beta_\perp^{uT}\nabla T_e - \beta^{uT}[h\times\nabla T_e]$ is the thermal force, S is the source density, $q = -\kappa_\parallel^e\nabla_\parallel T_e - \kappa_\perp^e\nabla_\perp T_e - \kappa^e[h\times\nabla T_e]$ is the heat flux, $\mathbf{h} = \mathbf{B}/B$, and expressions for the values of κ and β of a quiescent plasma are given in [3]. Equations (5) and (6) describe a complicated picture of the evolution of magnetic fields and the electron temperature, and must be solved numerically. The simplest nontrivial case in this case is the two-dimensional one.

4. We consider the system (5, 6) at $\Omega_e\tau_e \ll 1$. In this case we have

$$\frac{\mathbf{R}_T}{n} = -0{,}71\nabla T_e - 0{,}81\tau_e[\vec{\Omega}_e \times \nabla T_e],$$

$$q = -3{,}16\frac{nT_e\tau_e}{m}\nabla T_e - 5{,}7\frac{nT_e\tau_e^2}{m}[\vec{\Omega}_e \times \nabla T_e].$$

The equations for B and T_e take the form

$$\frac{\partial \mathbf{B}}{\partial t} + 0{,}81\operatorname{rot}\frac{\tau_e}{m}[\mathbf{B}\times\nabla T_e] = \frac{c}{e}[\nabla T_e \times \nabla \ln n], \quad (7)$$

$$\frac{3}{2}n\frac{\partial T}{\partial T} = 3{,}16\operatorname{div}\frac{nT_e\tau_e}{m}\nabla T_e - 5{,}7\operatorname{div}\frac{nT_e\tau_e^2}{m}[\vec{\Omega}_e \times \nabla T_e]. \quad (8)$$

Equation (7) describes the evolution of a magnetic field excited by the sources $(c/e)[\nabla T_e \times \nabla \ln n]$ and frozen into a liquid moving with velocity

$$\mathbf{u} = -0{,}81\frac{\tau_e}{m}\nabla T_e. \quad (9)$$

This drift of the magnetic field is connected not with the motion of the medium, but with the heat flux. It is the result of the fact that the magnetic field is frozen-in more strongly in the hot electrons than in the cold ones (this is apparently true also for a turbulent plasma). Under the conditions $l/a \gg \sqrt{m/M}$, the velocity of the thermal drift exceeds the hydrodynamic velocity.[1)]

5. The question of generation of magnetic fields is of great interest for spherical compression of a target by a programmed laser pulse, as proposed in [4]. Indeed, when magnetic fields with $\Omega_e\tau_e \sim 1$ appear, the electronic thermal conductivity decreases. The violation of the spherical symmetry of the thermal regime, due to the magnetic fields, may turn out to be even more significant. In the case of exact irradiation symmetry we have $\nabla T_e \parallel \nabla n$ and there are no magnetic fields. This state, however, can turn out to be unstable against perturbations of the magnetic field. Let us describe qualitatively the instability mechanism. We direct the x axis along $\nabla T_{0e}(\parallel \nabla n)$. A weak magnetic field $B \parallel Oz$ produces a Hall heat flow in the y direction. This flow leads to the appearance of a component of ∇T_e along the y axis. The additional temperature gradient together with the density gradient generates a magnetic field that strengthens or weakens the initial perturbation, depending on the sign of $(\nabla T_e \cdot \nabla n)$. Confining ourselves for simplicity to the case $ka \gg 1$ and $k \perp \nabla T_{0e}$, where k is the wave vector of the perturbation, we present the following formula for the increment (in the approximation in which $kl \ll 1$):

$$\Gamma = 1{,}8\frac{T_e\tau_e}{m}(\nabla \ln n \cdot \nabla \ln T_e). \quad (10)$$

In the case of spherical compression of the plasma in the zone of heat conduction (between the region of light absorption and the front of the heat wave) the density decreases, and the temperature increases in a direction away from the center of the sphere. There is consequently no instability in this case. The magnetic fields attenuate with drift to the center at a velocity $\mathbf{u}$ given by (9). An examination of the evolution of the magnetothermal perturbations against the background of a homogeneous magnetic field with $\Omega_e \tau_e > 1$ yields for Γ an expression that agrees with (10) in order of magnitude, but of opposite sign; the perturbations increase when $(\nabla T_e \cdot \nabla n) < 0$. This circumstance must be borne in mind when considering experiments with a laser plasma magnetized by an external magnetic field.

An important factor in the arguments advanced here was the inequality $l/a \gg \sqrt{m/M}$. In a laser plasma in the thermal conductivity zone, if the heat moves inward, satisfaction of this inequality is ensured automatically, since the plasma expands at a rate $\sim \sqrt{T_e/M}$, and the heat moves relative to the plasma with a velocity $\sim (l/a)\sqrt{T_e/m}$. It is worth noting that the presented analysis applies also to other (non-laser) plasmas if the indicated inequality is satisfied.

The authors thank S.I. Braginskii, E.P. Velikhov, M.A. Leontovich, and R.Z. Sagdeev for interest in the work and for remarks.

1) After the submission of this article, we learned of a calculation[5] of laser heating of a target, with allowance for the onset of magnetic fields. We note that this calculation does not take into account the thermal force, allowance for which leads to the effect described here.

[1] V.V. Korobkin and R.V. Serov, ZhETF Pis. Red. 4, 103 (1966) [JETP Lett. 4, 70 (1966)].
[2] I. Stamper *et al.*, Phys. Rev. Lett. 26, 1012 (1971).
[3] S.I. Braginskii, in: Voprosy teorii plazmy (Problems of Plasma Theory), M.A. Leontovich, ed., 1963, Vol. 1.
[4] E. Teller, IEEE Spectrum **10**, 60 (1973).
[5] N.I. Winsor and T.A. Tidman, Phys. Rev. Lett. **31**, 1044 (1973).