Contribution to the theory of the magnetic instability of a laser plasma

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It is shown that correct allowance for thermal diffusion changes the character of the excitation of the magnetic fields in a one-dimensionally inhomogeneous plasma; namely, the instability turns out to be convective and not absolute, while the magnitude of the excited fields is determined entirely by the density and temperature profiles of the plasma.

Experiments on laser heating of plasma[1] have revealed the generation of strong magnetic fields (on the order of several megagauss), which was attributed to the thermoelectric power produced in a plasma with crossed distributions of the density and temperature inhomogeneities. On the other hand, the possibility of magnetic instability of a one-dimensionally inhomogeneous plasma, in which the density and temperature gradients are parallel (a situation that can be realized when a plasma is heated by laser radiation) has been discussed in [2–4]. Using geometric optics for the description of the instability development, the authors of [2–4] have reached the conclusion that an inhomogeneous plasma with $\nabla T_0 \cdot \nabla n_0 > 0$ is unstable against exponential growth of magnetic fields, with increment $\gamma \approx 1.82 \chi \kappa_n \kappa_T$ ($\chi = T_0 \tau / m$ is the electronic diffusivity coefficient $\chi_n = \nabla \ln n_0$, and $\kappa_T = \nabla \ln T_0$).

In the present paper we solve the problem of magnetic instability of a one-dimensionally inhomogeneous plasma exactly, without the geometric-optics approximation. The solution obtained by us shows that the conclusion reached in [2–4], that the initial perturbations increase exponentially, is strictly speaking incorrect and is realized in exceptional cases. The perturbations actually do increase at $\nabla T_0 \cdot \nabla n_0 > 0$, but their growth is in general not exponential, being essentially determined by the density and temperature profile of the plasma. This behavior of the perturbations is due to the fact that the considered instability is not absolute in nature, but convective.

Just as in [2,3], the behavior of a fully ionized plasma will be investigated on the basis of the system of hydrodynamic equations[1)][5]

$$
\begin{gathered}
e\mathbf{E} + \frac{1}{n}\vec{\nabla} nT + 0{,}71\,\vec{\nabla} T + 0{,}81\,[\tau \boldsymbol{\Omega} \vec{\nabla} T] = 0, \\
\operatorname{div}\{n\chi \vec{\nabla} T + 1{,}82\, n\chi [\tau \boldsymbol{\Omega} \vec{\nabla} T]\} = Q .
\end{gathered}
\tag{1}
$$

Here $\boldsymbol{\Omega} = e\mathbf{B}/mc$ is the vector of Larmor rotation of the electron and Q is the source of plasma heating. The system (1) is valid under conditions when

$$
\left(\frac{c}{\lambda \omega_p}\right)^2 << \gamma\tau << \left(\frac{l}{\lambda}\right)^2 << 1,
\tag{2}
$$

where γ^{-1} and λ are the characteristic temporal and spatial scales of the process, $l = \sqrt{(T/m)}\,\tau$ is the electron mean free path, and ω_p is the average plasma frequency.

From the system (1) we get for the stationary state of an inhomogeneous plasma with $n_0(x)$ and $T_0(x)$, in the absence of an external field,

$$
T_0 \frac{d}{dx} \ln n_0 T_0^{1,71} = - eE_0, \qquad \frac{d}{dx}\left(n_0 \chi \frac{dT_0}{dx}\right) = Q(x).
\tag{3}
$$

Since the quantity $n_0 \chi$ does not depend on the density, the temperature distribution is determined entirely by the heat source $Q(x)$. At the same time, the electron density distribution, owing to the quasineutrality of the initial state, is determined by the ion density distribution, which can be maintained by some external forces (e.g., inertial forces produced during gas-dynamic expansion of the target). Thus, the first equation of (3) determines the field E_0 from the known $n_0(x)$ and $T_0(x)$.

For small perturbations of the initial stationary state, which depend on the time t and the coordinates x and z, the system (1) reduces to two equations

$$\frac{\partial\Omega}{\partial t} + \frac{\partial}{\partial x} u\Omega = \kappa_n \frac{\partial}{\partial z} \frac{\delta T}{m},$$
$$2{,}25u \frac{\partial}{\partial z}\Omega = -\left[\left(\frac{\partial}{\partial x} + \frac{5}{2}\chi_T\right)^2 + \frac{\partial^2}{\partial z^2}\right]\frac{\delta T}{m}, \qquad (4)$$

where $u(x) = -0.81(\tau/m)(dT_0/dx)$ is the thermodiffusion velocity of the electron. Assuming $\partial/\partial x \ll \partial/\partial z$, we obtain

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\delta T = \Gamma\delta T, \qquad \frac{\partial\delta T}{\partial z} = -2{,}25\,mu\Omega, \qquad (5)$$

where $\Gamma(x) = -2.25\kappa_n$ and $u = 1.8\kappa_n\kappa_T(T_0\tau/m)$. We note that the second equation of (5) determines the connection between Ω and δT at any instant of time, including $t=0$. Taking into account the definitions of $u(x)$ and $\Gamma(x)$, we can easily show that in (5) we have $u(\partial/\partial x) \gtrsim \Gamma$, and this determines the convective character of the instability. Moreover, the first equation of (5) can be reduced to the form

$$\frac{\partial\psi}{\partial t} + u\frac{\partial\psi}{\partial x} = 0, \quad \psi \equiv \ln[\delta T(x, z, t)n_0^{2.25}(x)]. \qquad (6)$$

It follows from (6) that the function ψ is constant on the characteristics, i.e., $\psi(z, x(x_0, t), t)_x = \psi(z, x_0)$, where x_0 is described from the characteristic equation $\int_{x_0}^{x} dx'/u(x') = t$.

Thus, the solution of (6) with the initial conditions $\delta T|_{t=0} = \theta_0(x, z)$ and $\Omega|_{t=0} = \Omega_0(x, z)$ can be represented in the form

$$\delta T(x, z, t) = \theta_0(x_0(x, t), z)\left[\frac{n_0(x_0(x, t))}{n_0(x)}\right]^{2.25}$$
$$\Omega(x, z, t) = \Omega_0(x_0(x, t), z)\frac{u(x_0(x, t))}{u(x)}\left[\frac{n_0(x_0(x, t))}{n_0(x)}\right]^{2.25}. \qquad (7)$$

According to (7), the behavior of the perturbations is essentially determined by the density and temperature profiles and in the general case is not characterized by an exponential growth. The conditions for the applicability of the analysis can then be written in the form $\kappa_n, \kappa_T \ll k_z$, where k_z is the wave number of the perturbations along the Oz axis.

It is easiest to trace the behavior of the perturbations in the case when the initial perturbations are homogeneous in x and the distribution of $n_0(x)$ and $T_0(x)$ is such that $(1/n_0)(dT_0^{5/2}/dx) = \text{const}$. Then $u(x) = \text{const}$ and $x_0 = x - ut$. In this case it follows from (7) that the initial perturbations increase at $dn_0/dx > 0$ if $dT_0/dx > 0 (u<0)$, and at $dn_0/dx < 0$ if $dT_0/dx < 0 (u>0)$, and the growth of the perturbations is due entirely to their drift over the plasma layer. This growth is exponential only in the particular case when the density has an exponential density profile, when $\kappa_n = \text{const}$.

The following are the conclusions of the preceding analysis:

1. The increase of the magnetic field in an inhomogeneous layer of a dense plasma is in general not exponential, and the field reaches a maximum on the periphery of the layer in the region where the plasma density is minimal.

2. The magnetic field on the periphery of the plasma layer can become stronger in comparison with the value inside the layer by more than two orders of magnitude when the plasma density changes by one order. If a field of 1 kG is produced inside the layer, it increases on the periphery to 10^5–10^6 G. Under the conditions of a laser plasma, a field of 1 kG can be easily generated in the plasma, for example by a slight ($\lesssim 1\%$) deviation from parallelism of the density and temperature gradients of the plasma,[1] or as a result of an initial temperature fluctuation $\delta T/T_0 \approx 10^{-2}$–$10^{-3}$.

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1) It is shown in [4] that there is no magnetic instability in the kinetic regime.

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