Close correlations in the Regge model

I. D. Mandzhavidze

Physics Institute, Georgian Academy of Sciences
(Submitted December 26, 1973)
ZhETF Pis. Red. **19**, 314–317 (March 5, 1974)

Starting from the unitarity condition and the conservation law for additive quantum numbers, equations are derived for the cross sections of multiparticle inclusive processes. Within the framework of the theory of complex angular momenta it is shown that, in the approximation in which the particle production is assumed to be independent, the broken symmetry of the exchange degeneracy is restored. The degree of breaking is determined by the value of the Pomeranchuk contribution.

The description of the differential cross sections $f_m^{(x)}(p_0, p_1, \ldots, p_{m+1})$ of m-particle inclusive processes with small values of m, or at $\bar{\eta} = (\xi/m) \gg 1$, where $\xi = \ln(s/s_0) \gg 1$, is confined to the principal contributions of the Regge model.[1] This approximation becomes, however, much less correct at $\bar{\eta} \sim 1$, owing to the need for correctly accounting for the proximity of the correlations in the space of the rapidities η.

The purpose of this paper is to call attention to the limitations on the vertex functions, describing the correlated production of particles at $\bar{\eta} \sim 1$, which can be obtained by starting from the unitarity condition and the conservation laws for additive quantum numbers. To this end we use the well known relations[1]

$$\sum_{n=0}^{\infty} z^n \sigma_n^{(x)}(\xi) = \sum_{m=0}^{\infty} (1-z)^m (-1)^m t_m^{(x)}(\xi) \equiv T^{(x)}(\xi; 1-z), \quad (1)$$

where $\sigma_n^{(x)}$ are the topological cross sections and

$$m!\, t_m^{(x)}(\xi) = \int \prod_{i=1}^{m} \frac{d^3 p_i}{2(2\pi)^3 \epsilon_i} f_m^{(x)}.$$

From (1) and the positiveness of $\sigma_0^{(x)}$ (i.e., the unitarity condition) it follows that $T^{(x)}(\xi; 1-z)$ should be an analytic function of z:

$$\sigma_0^{(x)}(\xi) \leqslant T^{(x)}(\xi; 1-z) \leqslant \sigma(\xi), \quad z \in [0,1], \qquad \sigma(\xi) = \sum_{n=0}^{\infty} \sigma_n^{(x)}(\xi). \quad (2)$$

By definition, which follows from (1), $\sigma_n^{(x)}$ describes the cross section for the production of n particles of sort x and an arbitrary number of particles of other sorts. By sort of particle we shall mean henceforth some aggregate, not necessarily complete, of internal quantum numbers, which makes it possible to separate the given sort of particles from the remaining ones. This generalization of the concept of sort of particles is permissible, so long as (1) remains unchanged regardless of the sort of particles detected in the given inclusive experiment. Accordingly, $\sigma_0^{(x)}$ describes cross sections without the production of particles of the given sort. Then, since $\sigma_0^{(x)} < \sigma$, this means that $T^{(x)}(1-z)$ should decrease with decreasing z, if $z \in [0, 1]$. This means, taking into account the positiveness of $t_m^{(x)}$, that there should be cancellation of different terms in the sum over m in (1), since the sum is of alternating sign. It is precisely this requirement which is reflected in the equations derived below. In terms of the Regge model, this cancellation causes the position of the singularities in the complex j-plane to shift to the left.

Having (1) and (2), it is easy to obtain the sought equations. Assume, for concreteness, the x denotes all the states characterized by a baryon charge $B = +1$. Then, if the initial state, say, is a two-nucleon state, then by virtue of the conservation of the baryon charge we have $\sigma_0^{(x)} = 0$ and, as follows from (1),

$$\lim_{z \to 0} T^{(B)}(\xi; 1-z) = 0. \tag{3}$$

By virtue of (2), this limit should exist.

Naturally, by changing the initial states it is possible to derive equations that complement (3).

Following the standard procedure (see, e.g., [2]), the correlations at large distances are described by Regge branch points, whereas at small distances they are described by pole contributions, including daughter contributions. Naturally, therefore, the contributions of the close and remote correlations to $f_m^{(x)}$ should satisfy (3) separately. We consider below only the pole contributions $P(\xi, 1-z)$ to $T(\xi, 1-z)$.

If we neglect the possible contributions of the exotic states with $B \geqslant 2$,[3] then the $\sigma_n^{(x)}$ should be described by the Poisson formula and $P^{(B)}$ can be represented in the form of a sum of contributions of Regge trajectories α_i:

$$P^{(B)}(\xi, 1-z) = \sum_i (g_i^{(B)})^2 \Theta_i \, e^{-[1-\alpha_i + (1-z)\Theta_i \Psi_i^{(B)}]\xi}, \tag{4}$$

where Θ_i is the signature factor of the ith trajectory, $g_i^{(B)}$ is the usual coupling vertex of the ith reggeon with the baryon. The constants $\Psi_i^{(x)}$ determine how many times, on the average, the particles of sort x are encountered when the ith reggeon is cut.

Let us examine what conditions are obtained for the constants in such a model of uncorrelated particle production. It is easy to see that (3) is satisfied if

$$\alpha_i - \Theta_i \Psi_i^{(B)} = \alpha_j - \Theta_j \Psi_j^{(B)}, \tag{5}$$

$$\Theta_i (g_i^{(B)})^2 = -\Theta_j (g_j^{(B)})^2, \tag{6}$$

where i and j take on the same values in both equations.

We consider first (6). If the Pomeranchuk singularity is a pole, then its contribution must be taken into account in (4). This in turn leads to the requirement that the exchange degeneracy symmetry be broken. Indeed, if the pomeron contribution is not taken into account in (4), then (6) is the usual condition that results from exchange degeneracy. Thus, as follows from (6), the contribution of the Pomeranchuk pole determines the degree of the exchange degeneracy symmetry breaking.

In addition to the condition $\sigma^{(B)} = 0$ we have at the given initial state the additional condition $\sigma_1^{(B)} = 0$. This leads, as can be easily seen, to the equation

$$\lim_{z \to 0} \left[\partial P^{(B)}(\xi; 1-z)/\partial z\right] = 0. \tag{7}$$

Hence, using (4), we obtain

$$\Theta_i \Psi_i^{(B)} = \Theta_j \Psi_j^{(B)}. \tag{8}$$

Further, comparing with (5), we see that

$$\alpha_i = \alpha_j, \tag{9}$$

which is valid only for exchange-degenerate trajectories. The fact that the exchange degeneracy symmetry is reconstructed by starting from the s-channel unitarity condition and the quantum-number conservation law seems interesting to us.

Thus, the presence of a Pomeranchuk pole leads to violation of the exchange degeneracy, meaning that it is necessary to take into account the close correlations regardless of the quantum numbers of the particles whose correlations are being studied. The degree of correlation undoubtedly depends on the quantum numbers, but it can be stated that such quantities as the topolotical cross sections $\sigma_n^{(x)}$ cannot be described by a Poisson formula if the Pomeranchuk singularity is a pole. We note here also, without proof, that to reconcile the solutions of Eqs. (3) and (7) it is necessary to take into account the "background" contributions from the remote correlations (more accurately, the regular contributions of the Regge branch cuts).

I am grateful to O. V. Kancheli for an interesting discussion of the results.

[1] A. H. Mueller, Phys. Rev. D 2, 2963 (1970).
[2] V. A. Abramovskii, V. N. Gribov, and O. V. Kancheli, Yad. Fiz. 18, 595 (1973) [Sov, J. Nuc. Phys. 18, No. 3 (1974)].
[3] J. R. Freeman and C. Quigg, CERN Preprint TH-1701.