Radiative corrections to leptonic decays of hadrons in renormalizable models of weak interactions

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It is shown that radiative corrections to the ratio of the vector constants of β and μ decays in renormalizable theories turn out to be essentially the same as in the usual scheme with the intermediate vector boson. Radiative corrections to π_{l2} decays are also considered.

Since the radiative corrections turn out to be finite in renormalizable theories of weak interactions, this raises the question whether a choice among different schemes can be made by investigations the contributions made by higher approximations to the amplitudes of leptonic decays of hadrons.

We begin with the question with the radiative corrections to the ratio G_β^v/G_μ^v of the vector constants of β and μ decays. As is well known,[1] in local four-fermion theory the electromagnetic correction go G_β^v/G_μ^v diverges logarithmically. Introduction of the intermediate vector boson makes this quantity finite even within the framework of the ordinary renormalizable theory. The cutoff parameter is then replaced by the W-boson mass.[2] It can be shown that the result does not depend on the magnetic moment of the W boson. The final expression takes the form (if we disregard the contribution of the soft quanta, allowance for which does not depend on the mode)

$$\frac{G_\beta^v}{G_\mu^v \cos\theta} - 1 = \frac{3\alpha}{8\pi}(1 + 2\bar{Q})\ln\frac{\mu_w^2}{m_h^2} \qquad (1)$$

Here θ is the Cabibbo angle, $\bar{Q}$ is the average charge of the isodoublet of the elementary fields that enter in the weak current, m_h is the characteristic hadron mass. In the derivation it is assumed that the mass of the W boson is much larger than all the hadron masses, $\mu_w \gg m_h$, and that at a virtual γ-quantum momentum $q \gg m_h$ the behavior of the amplitude $T_{\mu\nu}$ of the process $\gamma p \to W^+ n$ is determined by the commutator of the electromagnetic and weak currents. In other words, it is assumed that in this momentum region the strong interactions are inessential. Expression (1) contains an uncertainty $\sim\alpha/\pi$, since a determination of the exact value of m_h calls for detailed information on the amplitude $T_{\mu\nu}$ in the non-asymptotic region.

It is clear that in renormalizable theories the contribution of the virtual γ quantum to the ratio G_β^v/G_μ^v is also described by formula (1). In these theories it is necessary to account for, besides the electromagnetic field, also the contributions of the neutral vector field Z (in certain schemes) and of the scalar field σ, and also the corrections to the vertices $W_{\mu\nu}$ and W_{np} due to the W boson.

However, none of the additional corrections contain a large logarithm if one makes the natural assumption concerning the masses $\mu_w \sim \mu_z \sim m_\sigma$. Indeed, the logarithm $\ln(\mu_w/m_h)$ in expression (1) stems from the region of virtual momenta from m_h to μ_w. On the other hand, if the γ quantum is replaced by a particle with mass $\mu \gg m_h$, then the essential region is from μ to μ_w. Therefore all the new corrections do not exceed the uncertainty $\sim\alpha/\pi$ contained in formula (1).

More accurately speaking, the radiative corrections due to the σ field turn out to be much smaller, $\sim(\alpha/\pi)\times(m_h/m_\sigma)^2$, since the constants for the interaction between this fields and hadrons or leptons contain the ratios of the particle masses to μ_w.

As to the contributions of the W and Z bosons to the radiative corrections, they can be obtained by the same method[1] as the electromagnetic contribution. The correction to the vertices connected with the W boson turn out to be the same for the β and μ decays. Allowance for the Z boson in the Weinberg model[2–4] leads to the increment

$$\frac{3\alpha}{8\pi}(1 + 2\bar{Q})\ln\frac{\mu_z^2}{\mu_w^2} \qquad (2)$$

in the right hand side of (1). Thus, allowance for the Z

boson in this model leads only to the substitution $\mu_w \to \mu_z$ in formula (1).

Thus, a changeover to renormalizable theories has really no effect on the radiative corrections to G_β^v/G_μ^v.

We proceed to numerical estimates. Assuming $\sin\theta = 0.23 \pm 0.01$ and taking the contributions of the soft quanta into account, we obtain from a comparison of formula (1) with the decay data that

$$(1 + 2\overline{Q}) \ln \frac{\mu}{m_h} = 3{,}5 \pm 1{,}7. \qquad (3)$$

In the derivation of (3) we have used the formulas and data given in [1].

In the Georgi-Glashow model,[5] where $\overline{Q} = 1/2$, and for $\mu = \mu_w$ we frequently have the bound[6,7] 25 GeV $< \mu_w <$ 53 GeV, we find that the left-hand side of (3) lies in the interval (6.5—8) at $m_h = 1$ GeV. In the Weinberg model,[2-4] where $\mu = \mu_z > 75$ GeV and the quark charge is generally speaking not fixed, we obtain from expression (3) that $0.5 < \overline{Q} \leq 0.1$—$0.2$. The variant of the model[4] with integer quark charges ($\overline{Q} = 0.5$) does not agree very well with this limitation.

We consider now the radiative corrections to the $\pi(K) \to e\nu$ decays. They can turn out, generally speaking, to be significant, since the principal matrix element is proportional to the electron mass. Since the process proceeds with violation of γ_5-invariance, the greatest effect should be expected in the Georgi-Glashow model,[5] which contains heavy leptons. In this case an important role is played by diagrams with a neutral field σ, which can be subdivided into two groups. The first includes diagrams corresponding to a correction to the lepton vertex $W_{e\nu}$. The terms in this vertex which violate γ_5 invariance are proportional to the summary momentum of the leptons, with a factor m_x/m_σ^2 (or $1/m^x$, if $m_x > m_\sigma$). Here m_x is the mass of the heavy lepton.

The second group includes the diagrams that contain no W-boson pole. Their contribution, in the free-quark approximation, is

$$M = -\frac{G}{\sqrt{2}} \bar{e}(1+\gamma_5)\nu \langle 0 | \bar{p}\gamma_5 n | \pi \rangle \frac{\alpha}{16\pi} \frac{m_{q^+} + m_{x^+} + m_{q^0} m_{x^0}}{\mu_w^2 (m_\sigma^2 - m_{x^0}^2)}$$

$$\times \left(\frac{1}{m_{q^0}^2 - m_{x^0}^2} \ln \frac{m_{q^0}^2}{m_{x^0}^2} - \frac{1}{m_\sigma^2 - m_{q^0}^2} \ln \frac{m_\sigma^2}{m_{q^0}^2} \right). \qquad (4)$$

The subscripts x^+ and x^0 denote here the masses of the heavy leptons, while q^+ and q^0 label the hypercharge quarks. The hadronic matrix element can be estimated as follows:

$$\langle 0 | \bar{p}\gamma_5 n | \pi \rangle = \frac{i}{2m_p} \langle 0 | \partial_\mu a_\mu | \pi \rangle = \frac{1}{2m_p} f_\pi m_\pi^2, \qquad (5)$$

where a_μ is the axial-current operator, m_p is the p-quark mass, and f_π is the $\pi \to e\nu$ decay constant. Comparing the correction with the principal matrix element we find that the correction is small provided that m_q and m_x are not much larger than m_σ and μ_w.

Thus, one can hardly expect a large renormalization of the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ in the Georgi-Glashow model.

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