Effect of π condensate on the properties of nuclei

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It is shown that π condensation in heavy nuclei can lead to deformation of the nuclei and to the formation of shape isomers. The nucleon-density modulation due to the π condensate, and the associated effects in the case of scattering by nuclei, are considered.

As shown by one of the authors,[1–3] the square of the pion frequency becomes negative in nuclear matter, starting with a certain density n_c, and an electrically neutral π condensate is produced. We consider here π condensation in nuclei and effects associated with it. We shall speak, for the same of simplicity, of formation of a neutral condensate; the results can be easily generalized to include a three-component field. We use the pion units $\hbar = c = m = 1$.

The energy density of the ground state of the system in the presence of a static condensate field ϕ_0 was found in [2]:

$$E^{\pi}(\phi_0) = \sum_k \frac{\tilde{\omega}^2(k)}{2} \phi_{0,k} \phi_{0,-k} + \frac{\lambda}{4V} \int \phi_0^4 \, dV, \tag{1}$$

where $\tilde{\omega}^2(k) = 1 + k^2 + \Pi(0, k)$ and $\Pi(\omega, k)$ is the pion polarization operator. At a density close to critical, it suffices to expand $\tilde{\omega}^2(k)$ near the minimum, $\tilde{\omega}^2(k) = \omega_0^2 + \gamma(k^2 - k_0^2)^2$, and at $n > n_c$ we have $\omega_0^2 < 0$ and $\gamma > 0$. Changing over to the coordinate representation, we get

$$(\Delta + k_0^2)^2 f = \epsilon \left(f - \frac{4}{3} f^3\right), \tag{2}$$

where

$$f = \phi_0/a_0, \quad a_0^2 = 4|\omega_0^2|/3\lambda, \quad \epsilon = |\omega_0^2|/\gamma k_0^4 << 1.$$

Equations of the type (2) are solved by the asymptotic methods of the theory of nonlinear oscillations (see, e.g., [4]), viz., $f = \alpha f_0$, where f_0 is the solution of the linear equation ($\epsilon = 0$) and α is a slowly varying amplitude. The boundary conditions for Eq. (2) will be derived in a future communication. We indicate here that in the ϵ-approximation the energy of the system can be obtained in the main with the aid of the boundary condition $\alpha|_S = 0$. The remaining boundary conditions determine constants that are unimportant for the calculation of the energy under the conditions considered here.

For a spherical nucleus, Eq. (2) has a solution that goes over with increasing depth into spherical layers

$$f = \mathrm{th}\,\mu(R - r)\cos k_0 r, \tag{3}$$

$\mu = |\omega_0| 2\sqrt{2\gamma}k_0$, $\mu R >> 1$ with energy

$$E = E_0 V + E_S S, \tag{4}$$

where $E_0 = -\omega_0^4/\sigma\lambda$, $E_S = 4|E_0|/3\mu$, V is the volume of the nucleus, and S is the surface area. It is also easy to find a solution that goes over in the interior into plane layers

$$f = \mathrm{th}\,\mu(\sqrt{R^2 - \rho^2} - |z|)\cos k_0 z \tag{5}$$

at $\cos\theta \gg \sqrt{\epsilon}$, $\mu\sqrt{R^2 - \rho^2} \gg 1$, $\rho^2 = x^2 + y^2$. The energy corresponding to (5) is

$$E = E_0 V + 2E_S S_e, \tag{6}$$

where S_e is the area of the equatorial cross section. The expressions (4) and (6) for the energy are valid also for elliptic nuclei. It follows from the presented expressions that in heavy nuclei ($\mu R \gg 1$) the solution with plane layers yields a lower surface energy, i.e., (5) is more favored energywise than (3), whereas in lighter nuclei ($\mu R \sim 1$) a spherically symmetrical solution is obtained.

Since the condensate increment to the surface energy in the case of plane layers is proportional to the cross section of the nucleus, the presence of the condensate (5) contributes to the dilatation of the nuclei along the rotation axis. Let us examine the deformation of a spherical nucleus. We obtain the dependence of the energy of the nucleus on the quadrupole deformation parameter β.

$$E(\beta) = \frac{\alpha(\beta)\beta^2}{2} - \frac{4\pi R^2 E_S \beta}{3} \tag{7}$$

Account must be taken of the well known fact that the rigidity to small deformations ($\beta < A^{-1/3}$) is determined by the misalignment of the shell structure and is of the order of $\alpha(0) \sim \epsilon_F A$, whereas the rigidity with respect to large deformations is determined by the surface energy of the system and, as follows from the semiempirical formula $\alpha(\beta) \approx (1/6)\epsilon_F A^{2/3}$ for the binding energy of nuclei, we have $\beta > A^{-1/3}$, i.e., it is smaller less than $\alpha(0)$.

It is easy to see that the minimum of $E(\beta)$ corresponds to a state with small deformation $\beta_0 = 4\pi R^2 E_S/3\alpha(0)$, and that at $(2/3)\pi R^2 E_S > d(\alpha\beta^2)/d\beta|_{\min}$ a second minimum is produced on the $E(\beta)$ curve, corresponding to the so-called shape isomers (see, e.g., [5]). We note that if the second minimum exists as a result of on-shell factors, its condensate becomes deeper. However, a minimum can appear also when shell calculations do not call for it. We note also that the initial rigidity of the nucleus $\alpha(0)$ may turn out to be so large, that owing to the smallness of the equilibrium deformation β_0 the corresponding rotational band falls in the region of the single-particle energies, i.e., it becomes unobservable.

As noted earlier, [2] π condensation leads to modulation of the nucleon density. Far from the edge of the nucleus we have

$$n = n_0 + n_1 \cos 2k_0 z \tag{8}$$

for the case of plane layers and

$$n = n_0 + n_1 \cos 2k_0 r \tag{9}$$

for spherical layers. This modulation of the density can lead to a noticeable contribution to the cross section for scattering by nuclei at momentum transfers $q \sim 2k_0$. In the case of plane layers and scattering by non-oriented nuclei, the effect becomes somewhat weaker because of averaging over the directions of the layers. It should be noted in this connection that in a large number of recent experimental papers the electric form factors of the nuclei were determined as functions of the momentum transfer (see, e.g., [6—8]). In the region of $q \approx 2$—$3\ \mathrm{fm}^{-1}$ one observes systematic deviations from the form factors corresponding to a smooth density distribution. Attempts to attribute these deviations to shell fluctuations of the density have encountered serious difficulties (see, e.g., [9]). It is possible that these deviations are due to the condensate modulation of the density, which would lead to corrections to the form factor precisely at $q \sim 3\ \mathrm{fm}^{-1}$.

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