Thermodynamic method of calculating multiperipheral diagrams

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It is shown that the differential cross section corresponding to a multiperipheral diagram with exchange of Regge trajectories between heavy blocks can be represented in a form analogous to the microcanonical distribution in thermodynamics. The methods of thermodynamics can therefore be used quite effectively to calculate different distributions and fluctuations. By way of example, a diagram with π-exchange between irreducible blocks is considered. The total and topological cross sections are calculated.

Let us consider the multireggeon diagram shown in the figure, with exchange trajectories $\alpha_i(t_i)$ between the blocks. We assume that the amplitudes of the blocks are known and the interference between different blocks can be neglected. We shall show that the cross section corresponding to the multireggeon diagram can be easily calculated by methods developed in thermodynamics, provided the following condition is satisfied

$$s_i >> q_{\perp i}^2, \quad s_{i,i+1} >> s_i \cdot s_{i+1}, \tag{1}$$

where $q_{\perp i}$ is the transverse momentum of the ith block.

We shall show subsequently that this condition corresponds to the condition of statistical independence of a subsystem in thermodynamics.

Under condition (1), we can write

$$t_i = -r_i - \kappa_i^2, \quad i = 1, 2 \ldots n-1; \quad t_0 = t_n = \mu^2, \tag{2}$$

$$r_i = (s_{i+1} s_i)/s_{i,i+1}, \tag{3}$$

$$d\Phi = \frac{1}{s}\left\{\prod_{i=1}^{n} d\Phi_i\, ds_i\right\}\left\{\prod_{i=1}^{n-1}\frac{d\kappa_i^2}{4(2\pi)^3}\right\}\left\{\prod_{i=1}^{n-2} d\Delta y_i\right\}, \tag{4}$$

where κ_i is the transverse momentum of the ith reggeon, $d\Phi$ is the element of invariant phase space of all the particles, $d\Phi_i$ is the same for the particles produced in the ith block, and Δy_i is the difference between the rapidities of the blocks (see the figure).

The amplitude of the multireggeon diagram is equal to

$$U = \left\{\prod_{i=1}^{n} U_i(t_{i-1}, t_i, s_i, \Phi_i)\right\} \times \left\{\prod_{i=1}^{n-1} \eta[\alpha_i(t_i)][(s_0 s_{i,i+1})/(s_i s_{i+1})]^{\alpha_i(t_i)}\right\}, \tag{5}$$

where $U_i(t_{i-1}, t_i, s_i, \Phi_i)$ is the amplitude of the ith block, $\eta[\alpha_i(t_i)]$ is the signature factor, and s_0 is in general arbitrary. To simplify the notation, we take s_0 to be the unit of the energy squared.

The cross section corresponding to the multireggeon diagram is equal to

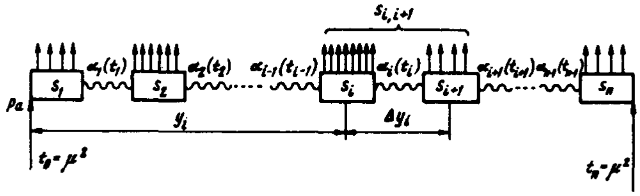
$$d\sigma = (|U|^2/2s)d\Phi = \left\{\prod_{i=1}^{n} \tilde{\sigma}_i(t_{i-1}, t_i, s_i) s_i\, ds_i\right\} \times \left\{\prod_{i=1}^{n-1} |\eta[\alpha_i(t_i)]|^2 \left(\frac{s_{i,i+1}}{s_i s_{i+1}}\right)^{2\alpha_i(t_i)} \frac{d\kappa_i^2}{2(2\pi)^3}\right\}\left\{\prod_{i=1}^{n-2} d\Delta y_i\right\}, \tag{6}$$

where

$$\tilde{\sigma}_i(t_{i-1}, t_i, s_i) = \int |U_i(t_{i-1}, t_i, s_i, \Phi_i)|^2/2s_i\, d\Phi_i. \tag{7}$$

We assume that the dependence of $\tilde{\sigma}_i$ on t_{i-1}, t_i, and s_i can be factorized, i.e.,

$$\tilde{\sigma}(t_{i-1}, t_i, s_i) = \bar{\sigma}_i(s_i)\,\phi_i(t_i)\,\phi_i(t_{i-1}). \tag{8}$$



In the literature it is customary to parametrize $\phi_i(t_k)$ in the form

$$\phi_i(t_k) = \exp\left(\frac{1}{2}\Lambda_i t_k\right). \tag{9}$$

Substituting (8) in (6) and integrating with respect to $d\kappa_i^2$, we arrive at the expression

$$d\sigma = \frac{1}{s^2}\left\{\prod_{i=1}^{n}\bar{\sigma}_i(s_i)s_i\,ds_i\right\}\left\{\prod_{i=1}^{n-1}g_i(r_i)\right\}\left\{\prod_{i=1}^{n-2}d\Delta y_i\right\}, \tag{10}$$

where

$$g_i(r_i) = \int_0^\infty \frac{d\kappa_i^2}{2(2\pi)^3}\left|\eta[\alpha_i(t_i)]\right|^2 r_i^{-2\alpha_i(t_i)}\phi_i(t_i)\phi_{i+1}(t_i). \tag{11}$$

We introduce new variables

$$\zeta_i = -\ln r_i, \qquad \xi_i = \ln s_i, \qquad \xi = \ln s. \tag{12}$$

It is easy to prove that

$$\sum_{i=1}^{n}\xi_i + \sum_{i=1}^{n-1}\zeta_i = \xi. \tag{13}$$

Formula (10) can therefore be written in the form

$$d\sigma = \frac{1}{s^2}\left\{\prod_{i=1}^{n}\bar{\sigma}_i(e^{\xi_i})e^{\xi_i}d\xi_i\right\} \times\left\{\prod_{i=1}^{n-1}g_i(e^{-\zeta_i})d\zeta_i\right\}\delta\left(\xi - \sum_{i=1}^{n}\xi_i - \sum_{i=1}^{n-1}\zeta_i\right). \tag{14}$$

The distribution (14) is perfectly analogous to the microcanonical distribution in thermodynamics (see, e.g., [1]), the role of the subsystem energies being assumed by ξ_i and ζ_i, while $g_i[\exp(-\zeta_i)]$ and $\bar{\sigma}_i[\exp(\zeta_i)]\times\exp(2\zeta_i)$ play the role of state densities. To calculate all the quantities of interest to us we can therefore use the mathematical formalism of thermodynamics, by introducing quantities analogous to those in thermodynamics. Although these quantities have utterly different physical meanings, we deem it expedient to designate them by the same letters and give them the names usually used in thermodynamics, adding the adjective "multiperipheral." Thus, the multiperipheral free energy is equal to

$$F = -T\left\{\sum_{i=1}^{n}\ln z_{1i} + \sum_{i=1}^{n-1}\ln z_{2i}\right\}, \tag{15}$$

where T is the multiperipheral temperatures, z_{1i} and z_{2i} are partition functions:

$$z_{1i} = \int e^{-(\xi/T)}\bar{\sigma}_i(e^{\xi})e^{2\xi}d\xi = \int\bar{\sigma}_i(s)s^{1-(1/T)}ds, \tag{16}$$

$$z_{2i} = \int e^{-(\zeta/T)}g_i(e^{-\zeta})d\zeta = \int g_i(r)r^{(1/T)-1}dr. \tag{17}$$

Let us find the entropy S

$$S = -dF/dt = \sum_{i=1}^{n}\{\ln z_{1i} + (\bar{\xi}_i/T)\} + \sum_{i=1}^{n-1}\{\ln z_{2i} + (\bar{\zeta}_i/T)\}, \tag{18}$$

where $\bar{\xi}_i$ and $\bar{\zeta}_i$ denote the mean values of ξ_i and ζ_i

$$\bar{\xi}_i = (1/z_{1i})\int(\ln s)\bar{\sigma}_i(s)s^{1-(1/T)}ds, \tag{19}$$

$$\bar{\zeta}_i = -(1/z_{2i})\int(\ln r)g_i(r)r^{(1/T)-1}dr. \tag{20}$$

T is a function of s and is determined by equating ξ to the multiperipheral energy:

$$E = F + TS = \sum_{i=1}^{n}\bar{\xi}_i + \sum_{i=1}^{n-1}\bar{\zeta}_i = \xi \equiv \ln s. \tag{21}$$

Finally, we obtain with the aid of (18) the cross section corresponding to the multireggeon diagram:

$$\sigma_n = s^{-2}(2\pi\overline{\Delta E^2})^{-1/2}e^S = \left[2\pi\left(\sum_{i=1}^{n}\overline{\Delta\xi_i^2} + \sum_{i=1}^{n-1}\overline{\Delta\zeta_i^2}\right)\right]^{-1/2} \times\left\{\prod_{i=1}^{n}z_{1i}\right\}\left\{\prod_{i=1}^{n-1}z_{2i}\right\}s^{(1/T)-2}. \tag{22}$$

Let us consider an example. Let all the trajectories be pions. We choose irreducible blocks, i.e., containing no π-exchange. The signature factor $\eta(\alpha_\pi)$ is defined somewhat differently than for the remaining trajectories

$$\eta(\alpha_\pi) = (\pi/2)\alpha'_\pi[i - \mathrm{ctg}(\pi\alpha_\pi/2)], \tag{23}$$

so that as $\alpha_\pi^1 \to 0$ it goes over into the propagator $(t-\mu^2)^{-1}$. For simplicity we assume all the Λ_i in (9) to be equal, $\alpha_\pi^1 = 0$, and neglect the square of the pion mass. Then $\bar{\sigma}_i(s)$ coincide with the cross sections corresponding to the irreducible blocks. We assume furthermore that

$$\bar{\sigma}(s) = As^{-\beta} \text{ if } s > 1, \qquad \bar{\sigma}(s) = 0 \text{ if } s < 1, \tag{24}$$

i.e., we take the threshold value of s to be unity. Calculation of the thermodynamic quantities yields

$$z_1 = A[(1/T) - 2 + \beta]^{-1}, \qquad z_2 = \frac{1}{2}(2\pi)^{-3}\Lambda^{1-(1/T)}T\Gamma\left(\frac{1}{T} - 1\right), \tag{25}$$

$$\bar{\xi}_i = [(1/T) - 2 + \beta]^{-1}, \quad \overline{\Delta\xi_i^2} = \bar{\xi}_i^2, \tag{26}$$

$$\bar{\zeta}_i = -\psi[(1/T) - 1] + T + \ln\Lambda, \quad \overline{\Delta\zeta_i^2} = \psi^1\left(\frac{1}{T} - 1\right) + T^2, \tag{27}$$

where ψ and ψ' are the Euler psi function and its derivative.

As shown by Ter-Martirosyan, multiperipheral diagrams without inclusion of pomeron exchanges lead to a cross section that decreases significantly only at very high energies, $\sim 10^{50}$ GeV. If we stipulate $\sigma_{\text{tot}} = \sum\sigma_n \to \text{const}$, then we can show that

$$A = 4(2\pi)^3 \Lambda \beta. \tag{28}$$

In the study of σ_{tot}, it is more convenient to assume the number of blocks to be variable. The application of the corresponding formalism of thermodynamics leads to

$$\sigma_{tot} = (e^2 \overline{\Delta E^2})^{-1/2} (\bar{n} - 1)\, z_2^{-1}\, s^{(1/T)-2}, \tag{29}$$

$$\overline{\Delta E^2} = \bar{n} \overline{\Delta \xi_i^2} + (\bar{n} - 1) \overline{\Delta \zeta_i^2}$$

$$+ (\xi - \bar{\xi}_i)^2 + (\xi - \bar{\xi}_i)(\bar{\xi}_i + \bar{\zeta}_i), \tag{30}$$

where $\bar{n}$ is the average number of blocks and is calculated from the formula

$$\bar{n} = 1 + [z_1 z_2 / (1 - z_1 z_2)]. \tag{31}$$

The formula for the multiperipheral temperature as a function of s is similar to (21)

$$\xi = \bar{n} \bar{\xi}_i + (\bar{n} - 1) \bar{\zeta}_i \quad . \tag{32}$$

The detailed results of the numerical calculations will be given in a separate article. Here we note one result: we choose $A = 150\ \mathrm{GeV}^{-2}$, $\Lambda = 1.5\ \mathrm{GeV}^{-2}$, $s_0 = 0.1\ \mathrm{GeV}^2$, and $\beta = 0.1$. Then we obtain for calculation in accord with (25)—(32): $\sigma_{tot}(0.1\ \mathrm{GeV}^2) = 150\ \mathrm{GeV}^{-2}$, $\sigma_{tot}(1.5\ \mathrm{GeV}^2) = 153\ \mathrm{GeV}^{-2}$, $\sigma_{tot}(60\ \mathrm{GeV}^2) = 163\ \mathrm{GeV}^{-2}$. We see that the cross section increases at finite energies, although it is asymptotically constant.

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[1]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964 [Pergamon, 1969].
[2]K. A. Ter-Martirosyan, Phys. Lett. **B44**, 179 (1973).