Quark model for currents and electromagnetic hadron interactions

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A description is obtained of the elastic eN and elastic $e+p\to e+\Delta^+$ (1236) scattering within the framework of the Markov–Yukawa oscillator quark model.

In[1] we used the representation that the relative coordinates ξ of quarks interacting via a four-dimensional oscillator potential are located on a hypersurface ξ, $p=0$ (the Markov-Yukawa conditions) connected with the moving hadron. The matrix element of the electromagnetic current of the hadrons takes in this approach the form[2]

$$\langle p', J'\lambda' | J_\mu(0) | p, J, \lambda \rangle$$

$$= \sum_{i=1}^{3} (L_0/L^2)^2 \int \delta\Big(\sum_{\nu=1}^{3} l^{(\nu)} - p\Big)\delta\Big(\sum_{\nu=1}^{3} l'^{(\nu)} - p'\Big)$$

$$\times \prod_{j\neq i} \delta[(l^{(j)} - p/3)p]\,\delta[(l'^{(j)} - p'/3)p']\,\delta(\mathbf{l}^{(j)} - \mathbf{l}'^{(j)} - a^{(j)}\mathbf{L})$$

$$\times \bar{\psi}'(l'^{(1)}, l'^{(2)}, l'^{3}, J', \lambda')$$

$$\times \hat{Q}^{(i)}\Gamma_\mu^{(i)}\psi(l^{(1)}, l^{(2)}, l^{(3)}, J, \lambda) \prod_{i=1}^{3} dl^{(i)} dl'^{(i)}, \qquad (1)$$

where m, M, p, and p' are respectively the masses and momenta of the initial and final states, $l^{(i)}$ and $l'^{(i)}$ are the momenta of the quarks in the initial and final states, J and J' are the spins and λ and λ' are the helicities of the initial and final hadronic states, $\hat{Q}^{(i)}$ is the charge operator of the ith quark, and

$$L = \sum_{i=1}^{3} \frac{l'^{(i)} + l^{(i)}}{2}, \qquad a^{(i)} = \frac{L(l^{(i)} - l'^{(i)})}{L^2}. \qquad (2)$$

The operator $\Gamma_\mu^{(i)}$ of (1) was chosen in[1] in the form $\Gamma_\mu^{(i)} = \gamma_\mu^{(i)}$. In this case the electromagnetic form factors of the nucleons turned out to be in satisfactory agreement with experiment in a wide range of q^2. The ratio of the magnetic moments is $\mu_p/\mu_N = -3/2$. However, the obtained absolute value of μ_p turned out to be barely a third of the experimental value.

This difficulty can be overcome by choosing the vertex operator $\Gamma_\mu^{(i)}$ of the quark-photon interaction in the form

$$\Gamma_\mu^{(i)} = \hat{l}'^{(i)}\gamma_\mu^{(i)} + \gamma_\mu^{(i)}\hat{l}^{(i)}. \qquad (3)$$

The operator follows from the equation for the bound states of a system of three quarks in first order in the coupling constant, when the electromagnetic field is turned on, in accord with the minimality principle. The equation is[2]

$$\{\hat{\partial}^{(1)2} + \hat{\partial}^{(2)2} + \hat{\partial}^{(3)2} - U\}\psi(l^{(1)}, l^{(2)}, l^{(3)}) = 0. \qquad (4)$$

The potential U, just as in the preceding paper, is represented as a sum of three two-particle potentials of the oscillator type. Equation (4) with such a potential yields for the hadron mass spectrum $M^2 = m_0^2 + N\Omega$, where N is the total number of excitations of the oscillator with frequency Ω. In this model, the particle mass spectrum is essentially degenerate.[4–6] Therefore, just as in[6], the parameter Ω will be regarded as an adjustment parameter for the description of real particles.

The use of the Markov-Yukawa conditions in the normalization condition

$$\int \bar{\psi}\psi\,\delta\Big[p - \sum_{i=1}^{3} l^{(i)}\Big]\delta[(l^{(1)} - p/3)p]\,\delta[(l^{(2)} - p/3)p]\,dl^{(1)}dl^{(2)}dl^{(3)}$$

lifts the infinite hadron-spectrum degeneracy that follows from (4) [see[7] for details).

1. Elastic form factors of the nucleon ($N\gamma N$ vertex). The wave functions in the oscillator relativistic quark model have been constructed, for example, in the Appendix of[4]. As before, we assume that the coordinate part of the nucleon wave function is the ground state of Eq. (4).[1]

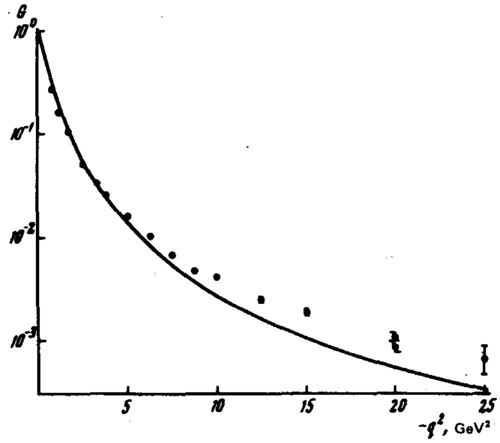

FIG. 1. Comparison of the theoretical form factor $G(q^2)$ with the experimental G_M^p/μ_p.[9]

Using the electromagnetic-vertex polarization given in[8] for a nucleon in the Breit system, we obtain

$$\left\langle \frac{1}{2};\frac{1}{2}\,|J_0|\,\frac{1}{2};\,-\frac{1}{2}\right\rangle = G_E\ ,$$

$$\left\langle \frac{1}{2};-\frac{1}{2}\,|J_-|\,\frac{1}{2};\,-\frac{1}{2}\right\rangle = -\sqrt{2}\,\frac{q}{2m}\,G_M$$

where $J_\pm = \mp(J_1 \pm iJ_2)/\sqrt{2}$. It follows from (1) that

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^N(q^2)}{\mu_N} = G(q^2), \quad G_E^N = 0, \tag{5}$$

with $\mu_p = 3$ and $\mu_N = -2$.

Thus, using the vertex (3), we actually obtain a proton magnetic moment close to the observed one. $G(q^2)$, first obtained in[1], was compared with experiment at Ω equal to the slope of the Regge trajectories, $\Omega \approx 1$ $(\mathrm{GeV/c})^2$. In view of the foregoing statements, we do not insist in the present paper on this condition, choose Ω unique for the description of the principal $SU(6)$ multiplet, and assume it equal to 0.78 $\mathrm{GeV}^2/\mathrm{c}^2$. The results of the comparison with experiment are shown in Fig. 1.

2. Let us consider the isobar production $e + p \to e + \Delta^+(1236)$. Assuming one-photon exchange, we parametrize the vertex $N\gamma\Delta^+$ (see Fig. 2), following[8], in the following manner:

$$\left\langle \frac{3}{2},\frac{1}{2}\,|J_0|\,\frac{1}{2},\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{10}}\,Q_2,$$

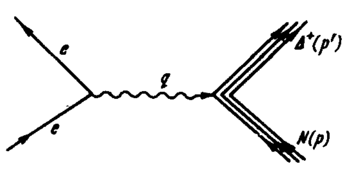

FIG. 2. Diagram of one-photon exchange for the electroproduction of the (1236) resonance.

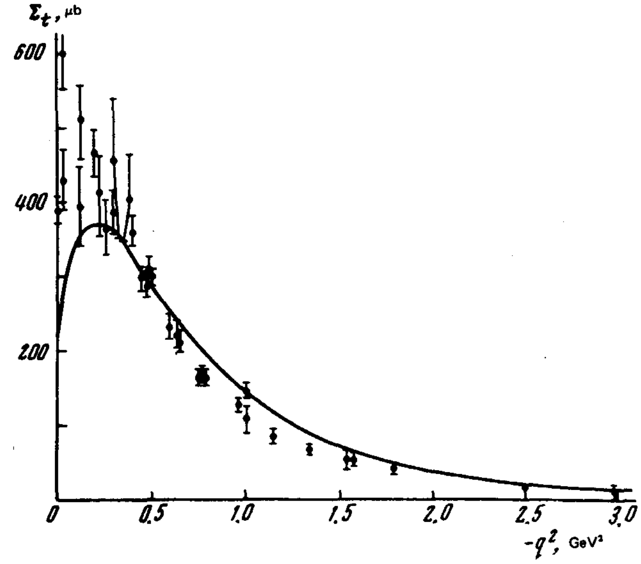

FIG. 3. Comparison of the theoretical value of $\Sigma_t(q^2)$ with the experimental data.[10,11]

$$\left\langle \frac{3}{2},\frac{1}{2}\,|J_+|\,+\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{2\sqrt{3}}M_1 - \frac{1}{2}\sqrt{\frac{3}{5}}E_2,$$

$$\left\langle \frac{3}{2},\frac{3}{2}\,|J_-|\,\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{2}M_1 + \frac{1}{2\sqrt{3}}E_2,$$

where $J_\pm = \mp(J_1 \mp J_2)/\sqrt{2}$.

In the oscillator quark model, the longitudinal and transverse electric components of the current are $Q_2 = E_2 = 0$; taking this into account, we obtain for the standard electroproduction cross section at $W^2 \equiv (p+q)^2 = M_\Delta^2$ [8]

$$\Sigma_t = \frac{4\pi\alpha}{3\Gamma k}M_1^2, \tag{6}$$

where Γ is the total width of $\Delta^+(1236)$, α is the fine-structure constant, and $k = (M_\Delta^2 - m^2)/2m$.

Calculation of the matrix element (1) with the wave functions ψ_p and ψ_Δ makes it possible to obtain the standard electroproduction cross section $\Sigma_t(q^2)$. Figure 3 shows a comparison of $\Sigma_t(q^2)$ at the value $\Omega = 0.78$ $(\mathrm{GeV/c})^2$ with the experimental data. As seen from the figure, the agreement with experiment ceases to be satisfied in the narrow region $0 \leq -q^2 < 0.1$ $(\mathrm{GeV/c})^2$. It seems to us that this discrepancy at small q^2 is due to a disruption of the oscillator interaction between quarks separated by a large space-time interval.

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[1]V. F. Dushenko, A. P. Kobushkin, and Yu. M. Sinjukov, Lett. Nuovo Cimento 8, 1 (1973).
[2]V. P. Shelest, Preprints ITP-67-51 and ITP-67-54, 1967.
[3]P. N. Bogolyubov, Elem. Chast. Atom. Yad. 3, 144 (1972) [Sov. J. Part. Nuc. 3, 71 (1973)].
[4]V. A. Matveev and R. I. Muradyan, JINR Preprint R2-3859, 1968; V. F. Dushenko, Ukr. Fiz. Zh. 14, 657 (1969).
[5]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[6]R.G. Lipes, Phys. Rev. D 5, 2849 (1972).
[7]V.F. Dushenko and A.P. Kobushkin, Preprint ITO-73-166E, 1973.
[8]N.R. Nath, Phys. Rev. D 7, 2046 (1973).
[9]D.H. Coward *et al.*, Phys. Rev. Lett. **20**, 292 (1968).
[10]A.B. Glegg, Internat. Symposium on Electron and Proton Interactions at High Energies, Liverpool, England, 1969., ed. by E.W. Braden and R.E. Rand.
[11]M. Breidenbach, Thesis, MIT Report No. MIT-2098-635, 1970.