Strong-interaction corrections to the muon anomalous moment and to the photon propagator

V. A. Petrun'kin and S. A. Startsev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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An empirical formula with asymptotic behavior $\sim(1/s)$ is obtained for the description of the total cross section of the $e^- - e^+$ annihilation into hadrons in the energy region beyond the ϕ resonance. The formula is obtained to calculate the corrections to the anomalous magnetic moment of the muon and to the propagator of the photon for the hadron polarization of the vacuum.

The justification and interpretation at experiments aimed at checking the applicability of quantum electrodynamics (QED) at high energies (short distances) depend to a considerable degree on the accuracy with which the contribution of the strong interactions to the measured quantities. Colliding-beam installations were recently used to measure the total cross section of the annihilation of an e^+e^- pair into hadrons at energies up to $2E = 5$ GeV.[1–4] The new data make it possible to refine the earlier calculations of the corrections that must be introduced into the anomalous magnetic moment (AMM) of the muon and into the photon propagator to allow for the hadronic polarization of vacuum (HPV). The correction to the muon AMM in the lowest order in α is given by[5]

$$\delta^{HPV} a_\mu = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \sigma(s) \int_0^1 \frac{x^2(1-x)}{x^2 + \frac{s}{m_\mu^2}(1-x)}\, dx\, ds . \qquad (1)$$

In (1), $\sigma(s)$ is the total cross section of the e^-e^+ annihilation into hadrons in the one-photon approximation, m_π is the pion mass, and m_μ is the muon mass. The propagator of a photon with momentum k is given by

$$D_{\mu\nu}(k^2) = \frac{1}{i}\left[\frac{\delta_{\mu\nu}}{k^2} - \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{\pi(-k^2)}{k^2}\right] . \qquad (2)$$

The HPV contribution to the correction to the free propagator, in the lowest order in α, is represented by[6]

$$\pi^{HPV}(-k^2) = -\frac{k^2}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} \frac{\sigma(s)}{s + k^2 - i\epsilon}\, ds . \qquad (3)$$

Earlier calculations with formulas (1) and (3) were performed in[5,7,8]. The scanty data on the cross section $\sigma(s)$ did not make it possible at that time to determine with any degree of reliability the parameters of the empirical formula used to describe the behavior of the cross section $\sigma(s)$ at $s = 4E^2 > 1$ GeV^2. In the present paper we describe the cross section of the e^-e^+ annihilation into hadrons, in the interval $4m_\pi^2 + s_0 = 1$ GeV^2. By the Breit-Wigner formulas, just as in[7,8], and in the region $s > s_0$ by the empirical formula

$$\sigma(s) = \sigma(s_0)\frac{s_0 - s_0'}{s - s_0'} , \qquad (4)$$

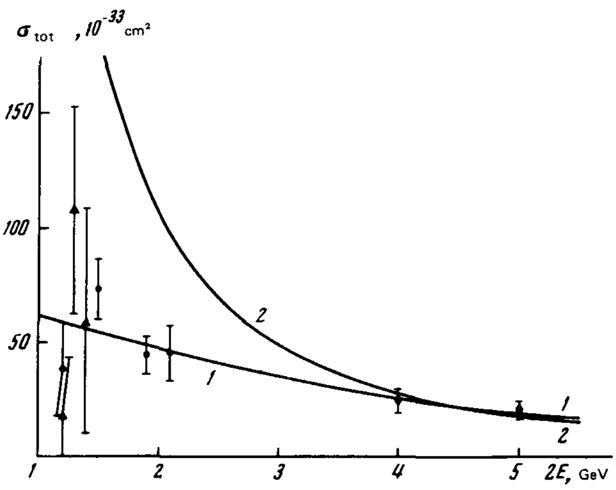
where the cross section $\sigma(s_0)$ at the point $s_0 \approx 1$ GeV^2 and s_0' are adjustment parameters. It is thus assumed that the cross section $\sigma(s)$ decreases like $1/s$ at large s. A similar prediction was obtained in the quark model.[9,10] The asymptotic $1/s$ behavior can be obtained also from simple dimensional considerations, by assuming that at large s there is no characteristic scale for the energy.[11] The figure shows the experimental data on the cross section $\sigma(s)$ in the region $s > 1$ GeV^2, and a plot of the function (4) (curve 1) with parameters $\sigma(s_0) = 61 \pm 9$ nb and $s_0' = -9.5 \pm 4$ GeV^2, obtained as a result of least-square fitting of (4). In the calculation of the parameters $\sigma(s_0)$ and s_0' the experimental points of[4] were left out, since they deviate greatly from all other data. Calculations of the correction to the AMM of the muon, using the vector-dominance model,[5] yielded a value $\delta^{HPV}\alpha_\mu = (6.4 \pm 0.5)\times 10^{-8}$. The contribution to $\delta^{HPV}\alpha_\mu$ from the region beyond the ϕ resonance was taken into account in[8,12], where the following values were obtained:

$(6.4 {}^{+2.4}_{-0.5})\cdot 10^{-8}$ [8] and $(6.8 \pm 0.9)\cdot 10^{-8}$ [12].

Our calculation yields[1)]

$$\delta a_\mu^{HPV} = (7.6 \pm 0.7)\cdot 10^{-8} . \qquad (5)$$

In (5) and in the table below we indicate only the errors that depend on the errors in the experimental data on the cross section $\sigma(s)$. The table (variant I) lists the values of the function $I(-k^2) = -2\,\mathrm{Re}\,\pi^{HPV}(-k^2)$, in terms of which we express the HPV corrections to the cross sections $d\sigma/d\Omega$ for the reactions $e^- + e^+ \Rightarrow e^- + e^+$ and $e^- + e^+ \Rightarrow \mu^- + \mu^+$ (see[8]). The HPV corrections to these cross sections are always smaller than or equal to $I(-k^2)$ and are consequently less than 5% in the $|k^2|$ interval from 1 to 10^2 GeV^2 expected to be accessible in



Total cross section for the annihilation of e^-e^+ pairs into hadrons: ●- Frascati,[1] ▲- Novosibirsk,[2] ◆- Cambridge.[3] Curve 1- $\sigma_{tot} = \sigma(s_0)(s_0 - s_0')/(s - s_0')$; curve 2 is from[13].

Values of the function $I(-k^2) \times 10^2$

$\sqrt{-k^2}$, GeV	0.7	1.4	3.0	5.0	6.0	7.0	10	100
Variant I	− 2.8 ±0.0	1.5 ±0.0	0.9 ±0.2	1.8 ±0.2	2.2 ±0.2	2.5 ±0.2	3.4 ±0.2	8.8 ±1.7
Variant II	–	–	–	3.0	3.3	3.5	4.1	7.6
$\sqrt{k^2}$, GeV	1.5	2.0	3.0	5.0	6.0	7.0	10	100
Variant I	1.2 ±0.0	1.5 ±0.1	1.9 ±0.1	2.6 ±0.2	3.0 ±0.3	3.2 ±0.3	3.9 ±0.4	8.8 ±1.7
Variant II	–	–	–	3.0	3.3	3.5	4.1	7.6

the nearest future. The maximum value of $I(-k^2)$ in this interval of $|k^2|$ does not depend strongly on the choice of the numerical values of the parameters $\sigma(s_0)$ and s_0'. Thus, the parameters $s_0' = -500\ \mathrm{GeV}^2$ and $\sigma(s) = 2\times10^{-32}$ cm^2 correspond to $I(-25\ \mathrm{GeV}^2) = 0.04$ and $I(-10^2\ \mathrm{GeV}^2) = 0.08$. The HPV correction to the AMM of the muon remains practically unchanged in this case: $\delta^{\mathrm{HPV}} = 7.5 \times 10^{-8}$. The values of $I(-k^2)$ at large $|k^2|$ likewise do not depend strongly on the cross section $\sigma(s)$ immediately beyond the ϕ resonance. The figure shows the cross section $\sigma(s)$ obtained by Sakurai,[13] who used an energy sum rule (see also[14]). The corresponding values of $I(-k^2)$ (variant II) at $|k^2| \geq 25\ \mathrm{GeV}^2$ differ by a factor 1.0—1.7 from the values of $I(-k^2)$ obtained with the cross section (4). The maximum value of $I(-k^2)$ is less than 0.05 in this case. We use as the disturbed model a QED model with a modified photon propagator, e.g., that of[15]. It is then easy to show that measurements of the cross section $d\sigma/d\Omega$ of the reactions $e^- + e^+ \Rightarrow e^- + e^+$ and $e^- + e^+ \Rightarrow \mu^- + \mu^+$ with 10% accuracy at an energy $2E = 10$ GeV can yield an upper bound on the fundamental length, $l \lesssim 6\times10^{-16}$ cm, at a confidence level 95%. It is assumed here that the usual radiative corrections to the cross section at an energy $2E = 10$ GeV can be calculated at the required accuracy, and the correction for weak interactions is still small.

[1)] Plans are underway at CERN to measure the muon AMM with accuracy $(1-2)\times10^{-8}$.

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