Feasibility of direct study of the character of strong interactions at short distances in experiments on $e^{\pm}e$ collisions. Role of quasilocal terms

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(Submitted February 18, 1974)
ZhETF Pis. Red. **19**, 413–418 (March 20, 1974)

The amplitude, which is determined only by the character of the interaction at short distances, can be measured directly in experiments on deep inelastic ee scattering (virtual forward $\gamma\gamma$ scattering). The form of the amplitude depends essentially on the character of the quasilocal (contact) terms. The possible form of the amplitude is discussed for models in which the strong interaction is turned off at short distances, and in a model in which this interaction is scale-invariant.

From experiments on $e^{\pm}e$ collisions one can extract directly information on the dependence of the $\gamma\gamma$ scattering cross sections on the effective mass of the produced particles $w=\sqrt{(q_1+q_2)^2}$ and on the photon masses q_1^2 and q_2^2 for both spacelike photons (the reaction $e^{\pm}e \to e^{\pm}e+h$ (hadrons, Fig. 1a, cf.[1]) and timelike photons at $q_1^2 > w_1^2 q_2^2$ (the reaction $e^+e^- \to \mu^+\mu^- h$, Fig. 1b, cf.[2]).

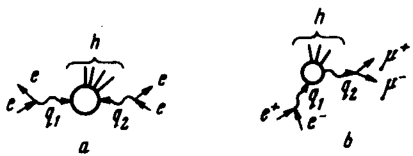

FIG. 1.

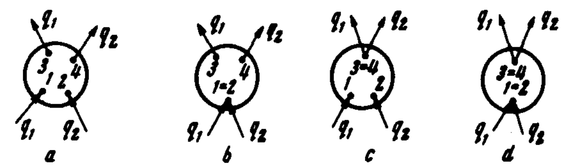

FIG. 2.

1. The $\gamma\gamma$ scattering cross sections measured in these processes are connected, by the optical theorem, with the absorptive part of the forward $\gamma\gamma$ scattering amplitude $W^{\mu\nu\mu'\nu'}(q_1^2, q_2^2, w^2, t=0)$, which is expressed in turn in terms of four electromagnetic currents

$$W^{\mu\nu\mu'\nu'}\delta(q_2 - q_2')$$

$$= \frac{1}{2(2\pi)^4}\int \prod_{i=1}^{4} d^4x_i\, e^{iq_1(x_1-x_3)+iq_2x_2-iq_2'x_4}$$

$$\times \langle 0|[T^*\{J^{\mu'}(x_1)J^{\nu'}(x_2)\}^*, T^*\{J^{\mu}(x_3)J^{\nu}(x_4)\}]_-|0\rangle. \tag{1}$$

The use of the symbol T^* is connected here with the usual ambiguity of the T-products at equal values of the arguments

$$\frac{1}{e^2}\frac{\delta^2 S}{\delta A_\mu(x)\delta A_\nu(y)} S^+$$

$$= T^*\{J^\mu(x)J^\nu(y)\} = T\{J^\mu(x)J^\nu(y)\}$$

$$+ V^\alpha(x)\delta(x-y)P_\alpha^{\mu\nu}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right);$$

$$P_\alpha^{\mu\nu} = \sum_{k,l=0}^{n} c_{kl}\left(\frac{\partial}{\partial x}\right)^k\left(\frac{\partial}{\partial y}\right)^l. \tag{2}$$

When the quasilocal terms VP are taken into account, the usual reduction to T-products of currents is impossible. In γp scattering, these terms contribute only to the unmeasurable real part of the amplitude, and in $\gamma\gamma$ scattering they exert a significant influence on the measured quantities. When they are taken into account, the measurable absorptive part in w^2 is described by diagrams of the type of Fig. 2.

The quasilocal operators V can be of two kinds.

Quasilocal operators of the first kind arise as increments to the usual electromagnetic interaction $J^\mu A_\mu$ and are necessary to ensure gauge invariance of the results in second order in e. They appear in theories with charged bosons, where the kinetic term is $\sim(\partial_\mu\phi)^2$ and the minimal electromagnetic interaction is of the form $eJ^\mu A_\mu + e^2V^{\mu\nu}A_\mu A_\nu$. [For example, for a self-acting pion field $V=\pi^*\pi$ and $P=g^{\mu\nu}$, in chiral theory $V=\pi^*\pi(1+\alpha^2\pi^2)^{-1}$ and $P=g^{\mu\nu}$, and in the Yang-Mills self-acting field theory $V - W^{*\alpha}W^\beta$, and $P_{\alpha\beta}^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta}$]. These terms are automatically taken into account when gauge-invariant expressions are written for the amplitude; they will not be discussed here.

Quasilocal operators of the second kind stem from interactions of the type

$$\pi^0 F^{\mu\nu}\tilde{F}^{\mu\nu}\left(V\sim\pi^0,\ P^{\mu\nu}\sim\epsilon^{\mu\nu\alpha\beta}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}\right),\quad \epsilon F^{\mu\nu}F^{\mu\nu}\Big(V\sim\epsilon,$$

$$P^{\mu\nu}\sim g^{\mu\nu}\frac{\partial^2}{\partial x_\alpha\partial y_\alpha} - \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\Big)\ f^{\mu\nu}F^{\mu\alpha}F^{\nu\alpha}\ \Big(V\sim f^{\alpha\beta},\ P_{\alpha\beta}^{\mu\nu}\sim g_{\mu\alpha}g_{\nu\beta}$$

$$\times\frac{\partial^2}{\partial x_\gamma\partial y_\gamma} + g_{\mu\nu}\frac{\partial^2}{\partial x_\alpha\partial y_\beta} - g_{\beta\nu}g_{\alpha\gamma}\frac{\partial^2}{\partial x_\gamma\partial y_\mu} - g_{\nu\alpha}g_{\beta\gamma}\frac{\partial^2}{\partial x_\nu\partial y_\gamma}\Big)\ \text{etc.}$$

The need for introducing these interactions was established in the study of the $\pi^0\to 2\gamma$ decay in the PCAC scheme (cf. [3]) and of the $e\to 2\gamma$ decay in the violated gauge invariance scheme[4] (triangular anomalies). These interactions are gauge invariant. However, it is impossible to write down a simple conservation law (such as current conservation) for the corresponding hadronic operators V. All that follows from gauge invariance is that $\partial_\mu V^\alpha(x)P_\alpha^{\mu\nu}A_\nu(x) = 0$.

We can separate from V^α various tensor operators, e.g., scalar, pseudoscalar, tensor, etc. When speaking of V, we have in mind henceforth one of them.

2. The essential advantage of the considered processes over all other known ones is that it is possible here to investigate directly the strong interaction at short distances under conditions when the interaction at large distances does not influence the amplitude. Indeed, according to[2,5], in the measurable absorptive part of the forward $\gamma\gamma$ scattering amplitude at

$$q_1^2q_2^2 \gg w^2m_0^2 = (q_1+q_2)^2m_0^2 \gg m_0^4;\quad -q_i^2 \gg m_0^2;$$

$$q_1^2\sim q_2^2 \gg w^2 \gg m_0^2;\quad m_0\sim 1\ \text{GeV}/c^2 \tag{3}$$

the main contribution to the integral (1) is made by the region of small values of all the distances $x_{ij}^2 = (x_i - x_j)^2$ in Fig. 2a the contribution of large distances is suppressed. In particular, at $-q_1^2 \sim -q_2^2 \gg m_0^2$ this region is given by[5]

$$|x_{12}^2|,\ |x_{34}^2| \lesssim \frac{2q_1q_2}{q_1^2q_2^2}$$

$$= \frac{w^2}{q_1^2q_2^2} - \frac{1}{q_1^2} - \frac{1}{q_2^2};\quad |x_{13}^2| \lesssim \frac{2q_1q_2}{w^2q_1^2};$$

$$|x_{24}^2| \lesssim \frac{2q_1q_2}{w^2q_2^2};\quad |x_{14}^2| \lesssim \frac{2q_1q_2}{w^2q_1^2q_2^2}(w^2 - q_2^2);$$

$$|x_{23}^2| \lesssim \frac{2q_1q_2}{w^2q_1^2q_2^2}(w^2 - q_1^2). \tag{4}$$

In different models of strong interaction at short distances it is possible to obtain rather definite predictions for the amplitude in the region (3).

3. From the algebra of bilocal operators on the light cone[5] and from the parton model (see the reviews[6]) it follows that the cross section of the $\gamma\gamma\to h$ transition in region (3) differs only by a factor from the cross section for the production of pointlike particles

$$\sigma_{\gamma\gamma\to h} = C_1\sigma_{\gamma\gamma\to\mu^+\mu^-} + C_2\sigma^{(\text{Born})}_{\gamma\gamma\to\pi^+\pi^-}$$

$$\sim f_0\left(\frac{q_1^2}{w^2}, \frac{q_2^2}{w^2}\right)\frac{1}{\sqrt{(q_1q_2)^2 - q_1^2q_2^2}};\quad C_1,\ C_2\sim\text{const}. \tag{5}$$

The corresponding forward $\gamma\gamma$ scattering amplitude is described by very simple diagrams such as in Fig. 3. (This conclusion can be easily understood by recognizing that in such models the strong interaction at short distances is essentially excluded.)

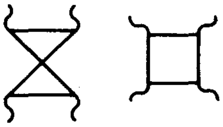

FIG. 3.

It should be noted that in the derivation of (5) it was tacitly assumed that the electromagnetic interaction is minimal, i.e., that there are no quasilocal terms of the second kind. However, remaining within the framework of the scheme, we can add such terms, e.g., $\sim \psi\gamma^5\psi F^{\mu\nu}\tilde{F}^{\mu\nu}$.[1)] Then the simplest diagram of Fig. 2 begins to predominate, and the cross section (for transverse photons) should increase like $w^2\sqrt{(q_1q_2)^2 - q_1^2q_2^2}$.

4. If the strong interaction at short distances is scale (and conformally) invariant, then the behavior of the amplitude is determined by the anomalous dimensionalities of the operators J and V (the anomalous dimensionality of an operator—its scale dimensionality—is canonical). As is well known, the anomalous dimensionality of the current is zero by virtue of its conservation. The anomalous dimensionality η of the operator V is generally speaking different from zero.

Taking this into account, the invariant amplitudes of the process should take in region (3) the form (apart from trivial factors)

$$f\left(\frac{q_1^2}{w^2}, \frac{q_2^2}{w^2}\right) + w^\eta \Gamma\left(\frac{q_1^2}{w^2}, \frac{q_2^2}{w^2}\right) + C w^{2\eta}. \qquad (6)$$

The first term corresponds here to the four-point function of Fig. 2a, the second to the three-point functions 2b and 2c, and the last to the two-point function 2d. [One cannot exclude the possibility of the second term of (6) vanishing by virtue of gauge invariance—cf.[7].] We thus have the following alternatives:

A. If $\eta > 0$, then at sufficiently large w the amplitude increases simply like $w^{2\eta}$ and does not depend on the photon masses q_i^2 (contribution of Fig. 2d). The first correction to this quantity (the contribution of 2b and 2c) is calculated in accordance with the well known rules.[7] The case $\eta \sim 0$ calls for a separate detailed analysis.

B. If $\eta \leq 0$ or if the quasilocal terms are insignificant for some reason, then the invariant amplitudes satisfy the simple similarity law[2)]

$$W = f\left(\frac{q_1^2}{w^2}, \frac{q_2^2}{w^2}\right), \qquad (7)$$

We emphasize that strong interaction at short distances, leading to scale invariance, should cause the ratio of $\sigma_{\gamma\gamma\to h}$ to the cross section for the production of a pair of pointlike particles not to be constant, in contradiction to (5). If the quasilocal term dominates at $\eta = 0$ because of a large numerical coefficient, then the amplitude is asymptotically constant.

5. In[10], a certain integral H of the cross section of the $\gamma\gamma \to h$ transition is expressed in terms of the anomalous constant S of the $\pi^0 \to 2\gamma$ decay and the quantity

$$R = \sigma_{e^+e^-\to h}/\sigma_{e^+e^-\to\mu^+\mu^-} \ (s\to\infty): \quad H = \frac{16}{3}S^2/R$$

namely, $H = (16/3)S^2/R$, assuming scale and $SU(3)$ symmetry at short distances. The derivation of this relation seems to be incomplete for two reasons: 1) it does not take quasilocal terms into account, 2) it remains unclear how it is possible (and if it is possible) to separate the constant H in actual fact. In addition, the relation between S and the π^0 lifetime is also the subject of a special hypothesis. All this does not make the relation $H = 16S^2/3R$ promising as a check on scale symmetry of the interaction at short distances.

I thank N. N. Achasov, V. M. Budnev, A. I. Vainshtein, V. G. Serbo, and I. B. Khriplovich for discussions.

1)The problem of the renormalizability of the obtained interaction and of the radiative corrections must be solved with allowance for the induced nonelectromagnetic interaction of the partons at short distances.

2)A relation of exactly this type is obtained from dimensionality considerations[8] at $|q_1^2|$, $w^2 \gg m_0^2$ and in particular at $w^2m_0^2 \gg q_1^2q_2^2$, where the similarity law (7) should be replaced in scaling theory by the relation $W = \psi[(w^2/q_1^2q_2^2)q_1^2q_2^2]$ (cf.[9]).

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