MAGNETIC TRAP FOR CURRENTLESS PLASMA WITH ELLIPTIC MAGNETIC SURFACES — STELLARON ("RING STELLARATOR")

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A stellarator trap is proposed with elliptic magnetic surfaces with semiaxes a and b; this trap has transport coefficients smaller by a factor $(b/a)^2$ than traps with round cross section. At equivalent plasma volumes, this makes it possible to increase the confinement time by a factor (b/a), reducing by the same token the corresponding summary energy ignition threshold and the energy release in the thermonuclear reaction. Estimates of the transport coefficients are obtained within the framework of the so-called "neoclassical" model of collision diffusion.

It is known that the condition of hydrodynamic stability in tokamaks, $q > 1$ (q is the stability margin), determines the maximum value of the current density in the plasma, and thus imposes a definite limit on the ohmic method of heating. To increase the limiting value of the current, a "ring type" tokamak was proposed, namely a tokamak with magnetic surfaces of elliptic cross section elongated along the principal axis of the torus [1].

In the present article we wish to call attention to the possibility of producing a trap of the stellarator type with magnetic-surfaces cross sections that are strongly elongated along the principal axis of the torus. We shall call this trap a "stellaron." Since the magnetic surfaces and the rotational transformation are produced in such traps not by the current flowing in the plasma but by external conductors, one should expect in similar systems to make the ratio of the semiaxes b/a as large as desired. We shall show that this makes it possible within the framework of the same approach as in the case of the ring tokamak, to decrease greatly the influence of the toroidal effects on the transport coefficients.

We consider by way of example a system with a magnetic field $\vec{B} = \nabla\phi$, described by a scalar potential

$$\Phi(u,v,s) = s + \epsilon c[Se_n(u,-q)se_n(v,-q)\cos nas - Ce_n(u,-q)ce_n(v,-q)\times \times \sin nas] \quad (1)$$

inside an elliptical cylinder $u < u_0$, where u and v are the elliptical coordinates in the cross section s = const, and s is the longitudinal coordinate along the cylinder axis [2]. The potential (1) corresponds to the lowest harmonic of a system of conductors in which the currents in the neighboring conductors flow in opposite directions, and the conductors are form helices

$$\frac{1}{n}\int_0^v (a_n + 2q\cos 2\xi)^{1/2} d\xi - as \cong \text{const} \quad (2)$$

on the elliptical cylinder $u = u_0$ (or on an elliptical torus in the approximation $a/R \ll 1$, where R is the principal radius of the torus). The functions $Se_n(u, -q)$, $Ce_n(u, -q)$, $se_n(v, -q)$, and $ce_n(v, -q)$ are the associated and ordinary Matthieu functions of integer order n of the parameter $q = n^2a^2c^2/4$ in MacLachlan's notation [3]. In accordance with the conditions under which the problem is considered (smallness of fast oscillations of the force lines relative to the averaged magnetic surfaces), the greatest interest attaches to the case when $a_n \sim n_2 \gg 2q \gg 1$, with $n > b_0/a_0 = 1/\sinh u_0 \gg 1$.

The equations for the magnetic force lines of the field described by the potential (1) can be solved by using the average method [4]. The equations for the force-line coordinates $\bar{u}$ and $\bar{v}$ averaged over the longitudinal current period $L = 2\pi n/a \lesssim /a_0$ have an integral

$$\bar{\psi}(\bar{u}, \bar{v}) = \bar{f}\exp[-(\text{ch}^2\bar{u} - \cos^2\bar{v})] = \text{const}, \quad (3)$$

which corresponds to the averaged magnetic surfaces $\overline{\psi}$. The function $\overline{f} = f(\overline{u}, \overline{v})$ in expression (3) is given by

$$f = Se_n(\bar{u})Ce_n'(\bar{u})se_n'(\bar{v})ce_n(\bar{v}) - Se_n'(\bar{u})Ce_n(\bar{u})ce_n'(\bar{v})se_n(\bar{v}), \tag{3a}$$

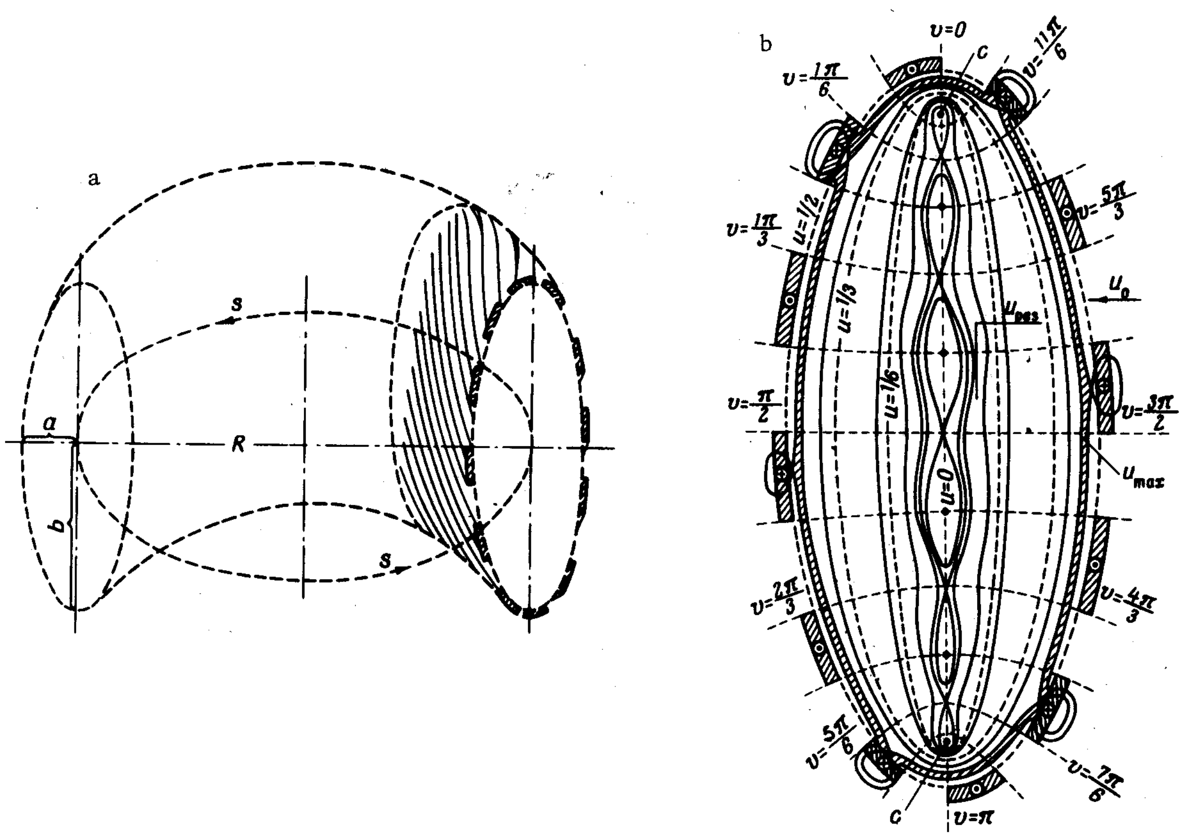
where the primes denote derivatives with respect to the arguments, and the dependence on the parameter "-q" has been omitted. The structure of the magnetic surfaces $\overline{\psi}$ = const, the diagram of the current system that produces the magnetic system, and the coordinate system, are all shown in Figs. a and b. A qualitatively correct description of the result of the structure of the surfaces $\psi(\overline{u}, \overline{v})$ = const can be obtained already in the extreme asymptotic limit $n \gg 1$, when $\overline{f}(\overline{u}, \overline{v}) \to (\cosh^2 nu - \cos^2 nv)$, and the current lines form helices $v - as$ = const.

The results show that it is possible, in principle, to produce a trap with magnetic surfaces that are strongly elongated in a vertical direction and have, on the average, an elliptic cross section. In addition, an inescapable result of such a deformation is also a one-dimensional resonantly-string-like structure near $u = 0$. However, the dimensions of this substructure are finite, and according to expression (3), their magnitude relative to u is given by

$$u \lesssim u_{\text{res}} \approx \frac{\text{ar sh } 1}{(a_n + 2q)^{1/2}} \sim \frac{1}{n}, \tag{4}$$

so that when $n > b_0/a_0 = 1/\sinh u_0$ no serious defects are produced in the magnetic structure. The horizontal aperture dimensions in terms of u, according to (5), can be of the order of u_0, if

$$\epsilon < \epsilon_0 \approx \frac{2ac}{n} e^{-nu_0} \tag{5}$$



Estimates at a suitable limitation of u (when nu $\sim$ 1) give acceptable values of the angle ι of the rotational transformation and of the shear, and an investigation of the optimization of the current system is beyond the scope of the present article. A more detailed exposition of this question will be presented in another article, where other methods of realizing traps of the stellatron type will be presented.

Let us estimate now the value of the diffusion coefficient D in the stellatron. This is easily done by using the model of Brownian motion of a trial particle, according to which $D \simeq \langle\Delta^2\rangle\nu$, where ν is the effective collision frequency, and Δ is the random deviation of the particle from the magnetic surface during the time between two successive collisions. Leaving out the simple calculations, we present an expression for the diffusion coefficient D_T connected with the presence of toroidality[1)]

$$D_{\mathrm{T}} \approx u^2 \varkappa \begin{cases} \dfrac{1}{\varkappa}\rho^2\nu, & \text{if } \dfrac{\varkappa v_{\mathrm{T}}}{R} < \nu \\[2ex] \dfrac{v_{\mathrm{T}}\varkappa}{R}\dfrac{\rho^2}{\varkappa^2}, & \text{if } \epsilon_{\perp}^{3/2}\dfrac{\varkappa v_{\mathrm{T}}}{R} < \nu < \dfrac{\varkappa v_{\mathrm{T}}}{R}, \\[2ex] \rho^2\nu\dfrac{\epsilon_{\perp}^{3/2}}{\nu^2}\dfrac{v_{\mathrm{T}}}{R^2} & \text{if } \epsilon_{\perp}\epsilon_{\perp}'\dfrac{\rho v_{\mathrm{T}}}{b} < \nu < \epsilon^{3/2}\dfrac{\varkappa v_{\mathrm{T}}}{R} \end{cases} \qquad (6)$$

where ν is the effective frequency of the electron-ion collisions, $\varkappa = \iota_{\nu}/2\pi$ is the total angle of rotational transformation normalized to 2π, v_T is the thermal velocity of the electrons, ρ is the Larmor radius, $\varepsilon_{\perp}$ is the ratio of the multipole "helical" component of the field to the longitudinal component B_s (i.e., $\varepsilon_{\perp} \sim \varepsilon ne^{nu}$), R is the principal radius of the torus, u = a/b, and $\varepsilon'_{\perp}$ is the derivative of $\varepsilon_{\perp}$ along the normal to the magnetic surface.

It follows from (6) that in the entire range of frequencies the diffusion coefficient (and also, as can be easily shown, the thermal-conductivity coefficient) is smaller for the stellatron by a factor $(b/a)^2$ than for the ordinary stellarator at equal angle of rotational conversion. Accordingly, the plasma lifetime in the stellatron is (b/a) times larger than in the ordinary stellarator.

Thus, if the theoretical prediction based on the existing concepts concerning transport phenomena in toroidal systems is correct, the summary energy threshold of ignition (and accordingly, energy output) of a thermonuclear reaction in a stellatron is smaller by roughly one order of magnitude (by a factor b/a) than in ordinary systems. On the other hand, within the framework of pure physical research, the construction of a stellatron would cast much light on the role of toroidal effects in plasma confinement. Of course, it is necessary first to investigate in sufficient detail the stability of the plasma in such devices, and to look into the problem of sufficient topological stability of the magnetic configuration.

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1)The total diffusion coefficient is $D_{cl} = D_T$, where $D_{cl} = \rho^2\nu$.

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