THEORY OF POMERON INTERACTION

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The theory of pomeron interaction is considered with the aid of the strong-coupling method [2] and the ε-expansion method [7].

The reggeon field theory [1] enables us to represent the hadron interaction amplitudes at ultrahigh energies in the form of a sum of reggeon diagrams, in which integrations are carried out with respect to the transverse momenta $k_{\perp i}$ transferred through the regions, and the logarithm of the energy ξ_i supplied to the given virtual reggeon. The parameter of this diagram expansion is the quantity $r^2\xi$, where ir is the reggeon interaction constant, $\xi = \ln E$, and E is the input energy.

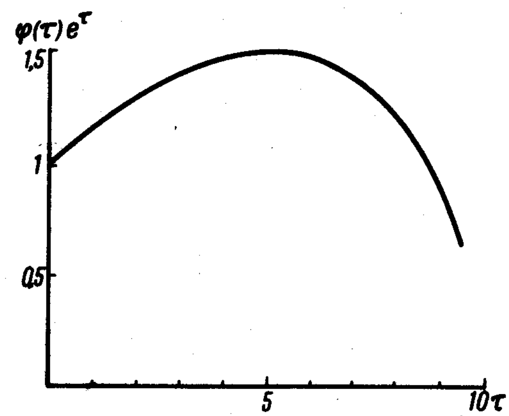
Gribov and one of the authors have shown [2] that two types of solutions are possible asymptotically at $r^2\xi >> 1$. One of them corresponds to the scale invariance of the pomeron

Green's functions, the pomeron field being ascribed an anomalous dimensionality determined by the interaction. The second variant, called weak coupling, corresponds to vanishing of the constant r and seemed to the authors of [2] preferable from the point of view of s-channel unitarity. A thorough study of this hypothesis (weak coupling) has shown that in this case four-reggeon interaction [4] and emission of particles by a pomeron [5] should also be forbidden, and all the interaction cross sections $\sigma_{tot}^{(i)}(s = \infty) = g_i^2$ of the different hadrons should be equal [6], etc. None of these hindrances follow from the theory in natural fashion. We have therefore investigated the forgotten alternative of strong coupling as $\xi \to \infty$ (see also [3]).

We made no assumptions whatever concerning the Reggeon constants, and applied to the reggeon Lagrangian Wilson's ε expansion [7], which is an effective numerical method accurate to within several per cent in the analogous problem of phase transitions.

In our case this method reduces to a generalization of the theory to a non-integer dimensionality $d = 4 - \varepsilon$ of the space of impact parameters, and expansion of the equations of the renormalization group in ε. In this article we summarize the principal results, and the details will be published later [3].

We have corroborated the scale invariance and have observed that it agrees with the unitarity conditions and with reggeon perturbation theory. The ε-expansion yields the following relations. The scattering amplitude $T(s, t)$ at the asymptotic energy is given by



$$\frac{1}{s}T(s, t) = i g^2 Z_0 \xi^{\eta}\phi(-R_0^2 t \xi^{\nu}), \quad \xi = \ln(s/s_0) \tag{1}$$

where

$$\eta = \frac{d}{2}\nu - 2\Delta = \epsilon/12 + 0(\epsilon^2) \cong {}^{1}/_{6}\ , \tag{2}$$

$$\nu = 1 + {}^{\epsilon}/_{24} + 0(\epsilon^2) \cong {}^{13}/_{12}, \tag{3}$$

$$\Delta = 1 - {}^{\epsilon}/_{4} + 0(\epsilon^2) \cong {}^{1}/_{2} \tag{4}$$

(Δ is the dimensionality of the pomeron field), Z_0 and R_0 are certain scale factors. The universal function $\phi(r)$ can also be expanded in powers of ε:

$$e^{r}\phi(r) = 1 + \frac{\epsilon}{12}\left\{e^{r/2} - 1 + \frac{r}{2}\ln\frac{2}{\gamma} + \left(1 - \frac{r}{2}\right)\int_0^{r/2}\frac{e^x - 1}{x}dx\right\} + 0(\epsilon^2). \tag{5}$$

Here $\gamma = 1.78$ is the Euler constant. This function is shown in the figure.

The total cross sections increase like

$$\sigma_{tot} = g^2 Z_0 \xi^{\eta}\ , \tag{6}$$

and the elastic cross sections decrease like

$$\sigma_{el} = \text{const}\ \xi^{-a}\ , \tag{7}$$

where

$$a = \frac{d}{2}\nu - 2\eta = 2 - \frac{7\epsilon}{12} + 0(\epsilon^2) \cong 5/6. \tag{8}$$

The inelastic amplitudes are characterized by an additional exponent δ, which is the dynamic dimensionality of the operator of the transition of two regions into particles, $U = \psi^{+}\psi \sim \xi^{-\delta}$. The corresponding cross sections for the production of n + 2 particles decrease like

$$\sigma_{n+2} = \text{const}\ \xi^{-a-n\beta} \tag{9}$$

with

$$\beta = 1 + \frac{d}{2}\nu - 2\delta \tag{10}$$

The ε-expansion yields for δ and β

$$\delta = 2 - \frac{\epsilon}{3} + 0\,(\epsilon^2) \cong 4/3, \tag{11}$$

$$\beta = 1 - {}^{\epsilon}/_{4} + 0\,(\epsilon^2) = 1/2. \tag{12}$$

The inelastic cross sections are much smaller here than in the multireggeon model [5], owing to the screening of the vertex of the particle emission by the pomeron.

The non-enhanced vertices N_n are also screened and lead to small corrections which do not give here for lack of space, and which are asymptotically inessential.

The suppression of the reactions with production of a definite number of particles means that the multiplicity increases with energy. Using the method of Abramovskii, Gribov, and Kancheli [8], we can find the renormalized spectrum and the renormalized multiplicity in the form

$$\frac{dN}{dy} = \rho_0 \frac{\sigma_{tot}(y)\sigma_{tot}(\xi - y)}{g^2 \sigma_{tot}(\xi)} \rightarrow \rho_0 Z_0 [\, y(1 - {}^{y}/_{\xi})]^{\eta}, \tag{13}$$

$$N = \int_0^{\xi} \frac{dN}{dy}\, dy = \rho_0 Z_0 \mathrm{B}(1 + \eta, 1 + \eta)\, \xi^{1+\eta}. \tag{14}$$

The first expression is valid accurate to decreasing non-enhanced corrections to σ_{tot}, while the second is valid only in the strong coupling limit, as $\xi \to \infty$. We now consider hadron scattering at the now existing energies.

It is possible in principle that strong coupling and scale invariance take place already at ISR energies, i.e., at[1] $\xi \sim 10$. It would be of great interest to compare the developed theory with the corresponding experimental data.

In this respect, however, we are pessimistic and believe that three-pomeron interaction is more likely to lead to small corrections at the attainable energies. Estimates [10] of the three-pomeron constant r on the basis of the ISR data [9] yield a value $r/\sqrt{\alpha'} \sim 1/10$, so that the expansion parameter $r^2/4\alpha'$ is smaller than the fine-structure constant.

The corresponding enhanced correction[1] to the total cross section increases 2% at the ISR energies.

The contribution of the non-enhanced graphs

$$\Delta_1 \sigma = \mathrm{Im} \cong - \frac{N_2^2}{2\alpha'\xi}; \qquad \Delta_2 \sigma = \mathrm{Im} + \mathrm{Im} \cong - gr \frac{N_2}{\alpha'} \ln \xi \tag{15}$$

can be predicted by using the value $N_2 \cong 1.3g^2$, which was obtained [11] for the NN interaction from data on diffraction production of particles, and $g^2 = 5(\mathrm{GeV/c})^{-2}$ [12].

The first term[2] $\Delta_1\sigma$ yields a 4% increase of σ_{tot} at ISR energies[1] $\xi \sim 10$, whereas the second term yields a like decrease of σ_{tot}, so that the effect vanishes.

Assuming that the remaining corrections are even smaller, we arrive at a 2% increase of σ_{tot} at ISR energies, owing to the enhanced correction, whereas the latest data yield a 10% increase. We hope that the subsequent experiments will make the situation clearer. We note also that our estimates of the non-enhanced diagrams at contemporary energies are not quite rigorous, since we do not know the multireggeon interaction constants.

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[1] We used throughout $\xi = \xi_0 + \ln(s/s_0)$, where $\xi_0 = R^2/\alpha'$ takes into account the finite radius of the pomeron vertex; here $s_0 = 2\ \mathrm{GeV}^2$, and for the NN interaction [12] we have $R^2 = 1.9 \pm 0.3$ and $\alpha' = 0.4$ in $(\mathrm{GeV/c})^{-2}$.

[2] Together with the analogous contribution of the multipomeron non-enhanced graphs [10].

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ERRATA

In the article by V. A. Gribkov et al., Vol. 18, No. 9, p. 319, the numerator of the fraction in Eq. (1) should read "$2I_0$" and not "$2T_0$."

In the article by Yu. I. Abrashitov et al., Vol. 18, No. 11, the following misprints should be corrected:

1) on p. 396, the time markers in Fig. 1 should be tagged "50 nsec" and "100 nsec," not "50 msec" and "100 msec,"

2) on p. 396, line 30, read "The dependence of the per unit length energy content" instead of "The dependence of the per unit energy content,"

3) on p. 397, in the caption of Fig. 3, add: "c - plasma, $n \approx 5\times10^{13}$ cm^{-3},"

4) on p. 397, line 11, read: "At large plasma densities ($n > 10^{13}$ cm^{-3})" instead of "At large plasma densities ($n \approx 10^{13}$ cm^{-3})."