Evolutionality and width of shock waves

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It is shown that the width of a shock wave is determined by the imaginary part of the wave vector of the rapidly damped waves that diverge away from the discontinuity.

As is well known, evolutionality of a shock discontinuity means the existence and uniqueness of a solution of the small-perturbation problem,[1,2] i.e., by specifying the amplitude of the wave incident on the discontinuity it is possible to determine uniquely the amplitudes of the outgoing waves. These are determined from $n-1$ linear equations, where n is the number of conservation laws on the discontinuity, while one equation eliminates the amplitude of the perturbation of the discontinuity itself.

If we neglect dissipation, then the number of waves of small amplitude traveling away from fast and slow shock waves is equal to $n-1$ (six) and these discontinuities are evolutional.

When account is taken of all the dissipative effects, the number of possible small-amplitude waves propagating in the plasma almost doubles: instead of two Alfven waves and two fast and two slow magnetosonic waves propagating in opposite directions, as well as one entropy wave (making a total of seven) there appear 13 different waves on each side of the discontinuity. The number of waves traveling away from the discontinuity also increases.

In the absence of degeneracy, i.e., if the plasma velocity component normal to the discontinuity does not coincide with the characteristic (all the conclusions are valid also for degenerate evolutional discontinuities), all 13 types of waves with small real frequency ω can be broken up into two groups: the first consists of seven traveling damped Alfven, entropy, and magnetosonic waves, the real part of the wave vector k of which is proportional in first-order approximation to ω, and the imaginary part to ω^2, and the second group consists of eight rapidly damped waves, in which Re $k \sim \omega$, and Im k is independent of ω in first order. The damping coefficients (Im k) of the waves of the first and second group contain the dissipation coefficients raised to the first and to the minus first powers, respectively. Therefore, if we let the dissipation coefficients tend to zero, the waves of the second group vanish, while those of the first group are transformed into undamped waves, which determine the evolutionality in the absence of dissipation.[3]

The evolutionality method is applicable only when the width L of the discontinuity can be neglected, i.e., if $|k| \ll 1/L$ for the incident wave and for all the waves that diverge outside the discontinuity front. If L is finite, this condition can be satisfied for all waves of the first group, by choosing sufficiently small ω; for the waves of the second group this cannot be done, since there Im k does not depend on ω.

On the other hand, a definite number of diverging waves should exist outside the front of an evolutional discontinuity having a stationary structure; these are traveling waves that do not vanish in the absence of dissipation. Consequently, the extra diverging rapidly-damped waves should attenuate inside the discontinuity and should be absent outside its front.

Thus, we have for the width of the discontinuity

$$L \gtrsim 1/\min |\mathrm{Im}\, k|, \tag{1}$$

where (1) is satisfied in order of magnitude, and the minimum is sought only among the diverging rapidly-damped waves.

By way of example we present a more detailed investigation of shock waves in gasdynamics.

In a gas, waves of the type $\exp[-i(\omega t - kx)]$ exist on both sides of the discontinuity, and have the following dependence of k on ω:

$$k_{1,2} = \frac{\omega}{V \pm c}; \quad k_{3-5} = \frac{\omega}{V}; \quad k_{6,7} = -i\frac{V}{\nu};$$
$$k_{8,9} = -i\frac{V^2(\chi + \zeta + {}^4/_3\nu) - c^2\chi/\gamma \pm \sqrt{B}}{2V\chi(\zeta + {}^4/_3\nu)}, \tag{2}$$

where

$$B = [V^2(\chi + \zeta + {}^4/_3\nu) - c^2\chi/\gamma]^2 - 4V^2\chi(\zeta + {}^4/_3\nu)(V^2 - c^2)$$

is essentially a positive quantity, $V > 0$ is the x-component of the gas velocity, c is the speed of sound, γ, ν, ζ, and χ are respectively the adiabatic exponent, the first and second kinematic viscosities, and the diffusivity. We confine ourselves to the first nonvanishing term of the expansion of k in powers of ω.

Waves with $k_{1,2}$ are acoustic; those with k_{3-5} are the entropy waves of the velocity-curl perturbation. The waves with k_{6-9} are rapidly damped. The direction of propagation of the rapidly damped waves can be assessed from the value of Im k, namely, in a stable homogeneous medium all waves are damped in the propagation direction, so that if Im $k > 0$ then the wave propagates in the positive direction of the x axis, and vice versa.

In the case of normal incidence on a shock wave, in front of which $V_1 > c_1$, behind which $V_2 > c_2$, and the plane of which is perpendicular to the x axis, we have the following diverging rapidly-damped waves: upstream waves with k_{6-9}, and a downstream wave with k_9. It can be shown that min |Im k| will belong to waves with k_9 on one side of the discontinuity or the other. We thus have ultimately

$$L \gtrsim \max | 2V_{1,2}\chi(\zeta + {}^4/_3\nu) : \{V_{1,2}^2(\chi + \zeta + {}^4/_3\nu) - c_{1,2}^2[\chi/\gamma] - \sqrt{[V_{1,2}^2(\chi + \zeta + {}^4/_3\nu) - c_{1,2}^2\chi/\gamma]^2 - 4V_{1,2}\chi(\zeta + {}^4/_3\nu)(V_{1,2}^2 - c_{1,2}^2)}\}|.$$

For weak shock waves $(V_1 - c_1)/V_1 \ll 1$ and we get $L \gtrsim [(\gamma - 1)\chi/\gamma + \zeta + 4\nu/3]/2(V_1 - c_1)$ in accord with[4].

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