Electromagnetic radiation of a charge in a Kerr field

I. M. Ternov, V. R. Khalilov, G. A. Chizhov, and I. I. Maglevannyĭ

Moscow State University

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A detailed analysis is presented of the properties of the electromagnetic radiation coming from a charge moving along a circular geodesic line in the equatorial plane of a Kerr field. At certain values of the parameters, the results are applicable to a Schwarzschild field.

We consider in this article the electromagnetic radiation of a charge moving along a circular geodesic line in a Kerr field.[1] The properties of the radiation, just as in flat space, depend essentially on the total energy of the charge.

We investigate two types of electromagnetic radiation": 1) radiation from a charge moving on a stable circular geodesic line ($E/\mu c^2 < 1$, E is the total energy and μ is the particle mass), and 2) synchrotron radiation proper[2] emitted by the charge as it moves along a relativistic geodesic line ($E/\mu c^2 \gg 1$).

A. Nonrelativistic source

If the circular velocity of the particle $\omega_p = d\phi/dt$ satisfies the inequality $\omega_p < [\Omega = \alpha/2(1+\sqrt{1-\alpha^2})$ is the angular velocity of the hole, M is the mass, and α is a parameter connected with the angular momentum J of the hole: $J = \alpha M^2$[1]], then the waves propagating towards the absolute horizon of the events x_{hor} (the surface of the unilateral value) can, according to[3], become amplified by "reflection" from the hole. The total radiation power reaching an observer at infinity can be represented in the form of a sum of two parts

$$W_c = W_{ch} + W_h, \tag{1}$$

where W_{ch} and W_h are the radiation powers of the charge and of the hole, respectively. The ratio

$$\frac{W_h}{W_c} = k(x_p) = \frac{4b(x_p)}{(1+b(x_p))^2} f(x_p) \tag{2}$$

will be called the "radiation gain" for each harmonic of the fundamental frequency, where

$$b = (\omega_p \delta)^{2l+1} 2^{2l} Q \prod_{n=1}^{l}\left(1 + \frac{4Q^2}{n^2}\right)\left[\frac{l!(l-1)!(l+1)!}{(2l)!(2l+1)!}\right]^2$$

$$\omega_p = \frac{1}{x_p^{3/2} + a}, \quad \delta = 2\sqrt{1-a^2}, \tag{3}$$

$$Q = \frac{2x_{hor}}{\delta}(\Omega - \omega_p), \quad x_{hor} = 1 + \sqrt{1-a^2},$$

and the function $f(x_p)$ is expressed in terms of solutions of the corresponding wave equations. The charge radiation power is equal to

$$W_h = W_c(1-k), \tag{4}$$

where k can be either positive or negative. The gain (2) depends on the location of the source, or more accurately on the distance from x_p to x_{hor}. From the point of view of the physics of the phenomenon, this result is explained by the fact that only waves incident on x_{hor} are amplified, and since the source is always inside a potential barrier for the radiation, the distance from the source to x_{hor} is appreciable. The "waves" propagate to x_{hor} and to $+\infty$, under the barrier up to the turning points x_1 and x_2 ($x_2 > x_1$) at which $\omega^2 = u_{eff}(\alpha, r, l)$.

It is clear therefore that at different source locations the value of k can vary, since the amplitudes of the amplified waves will be different. In particular, if x_p lies near the turning point x_2 and $x_p \gg x_{hor}$, then $f(x_p) \sim 1/4$ and the value of k is determined by the value of b. We note here that b depends on the character of particle motion. If the particle moves along a geodesic line in the given field, then $b \sim 1/(x_p^{3/2})^{2l+1}$ and decreases monotonically with increasing x_p. For non-geodesic circular value of k can become large.

Let us consider some limiting cases.

1) Let $x_p \gg 1$ (region of strong gravitational field). Then the radiation occurs at the fundamental tone, and the power is equal to

$$W_c = \frac{2}{3}\frac{e^2}{M^2}\frac{x_p^2\omega_p^2}{(1-b)^2}\left(1 - \frac{3}{4x_p}\right). \tag{5}$$

Here e is the charge of the particle. The factor $(1-b)^{-2}$ characterizes the effects of amplification ($b > 0$) and attenuation ($b < 0$) of the waves when they are reflected from the hole.

2) If the charge is near the absolute horizon of the events of the "black hole," having the maximum possible angular momentum, $\alpha \to 1$, then its angular velocity is $\omega_p = 1/2 - \epsilon$, $\epsilon \sim (x_p - 1) \ll 1$, $\Omega = 1/2$. We then obtain

$$W_c = \frac{15\epsilon^4}{(1-b)^2} I, \quad I = \frac{2}{3}\frac{e^2}{M^2}\omega_p^{8/3}. \tag{6}$$

I is the intensity of the dipole radiation of a particle rotating with angular velocity ω_p about a Newtonian center of mass M. In this region, an appreciable part of the radiation is captured by the hole.

Particular interest attaches to the question of the influence of the "reaction forces" of the radiation on the radiating particles. In the analysis of these forces, an important factor is that when $\Omega > \omega_p$ the phase velocity of the wave "propagating" in the system of an observer rotating together with the hole, from the particle to x_{hor}, is directed towards x_p for a remote observer[2)] (region $x < x_p$), i.e., this wave can perform positive work on the particle. If this part of the work cancels out the "negative" work of the radiation-reaction forces, which is due to the radiation of the wave "diverging" in the region $x > x_p$, then the motion of the particle along the given orbit will be stable (floating orbit). [5] If the particle radiates only at the fundamental tone and is in the region $1 < x_p < 3$, then as $\alpha \to 1$ there exists a floating orbit. The radius of this orbit is obtained from the condition $W_{ch} = 0$, which yields tentatively $x_p^f \sim 2$.

B. Relativistic source

If $E/\mu c^2 \gg 1$, then the circular orbits of the particle lie near the optical-geodesic orbits. Let us consider orbits with $x_p = x_0(1+\Delta)$, $\Delta \ll 1$ (x_0 is the radius of the optical geodesics). The main contribution to the intensity of the radiation is made in this case by higher harmonics of the fundamental: $\omega = m\omega_p$, $m \gg 1$. On all relativistic orbits we have $\omega_p > \Omega$, i.e., the "hole" cannot amplify the waves radiated by a particle.

Geodesic electromagnetic synchrotron radiation (GSR) is characterized by the following properties: 1) the spectral and angular distributions of the GSR intensities corresponding to the σ and π components of linear polarization[2] are given by

$$W_{\sigma,\pi} = \frac{1}{32\pi^{3/2}}\frac{\theta^2}{M^2}x_0(x_0 - 1)$$

$$\times e^{-C(x_0) - \frac{\pi}{2}\left\{\frac{m}{m_0} + [2(l-|m|)+1]\tilde{\mu}\right\}} C_{\sigma,\pi}\,, \qquad (7)$$

$$C_\sigma = \frac{8\cdot 3^{1/4}}{(3+x_0)^2\omega_0^{1/2}}$$

$$\times\left|\Gamma\left(\frac{1}{4} + \frac{im}{2m_0} + \frac{i3\tilde{\mu}}{2}\right)\right|^2 2m^{1/2}\pi^{-1/2}\sin^{2(m-1)}\theta$$

$$\times(1+\cos^2\theta)\,\delta_{e,m}\,,$$

$$C_\pi = 3^{1/4}\omega_0^{1/2}$$

$$\times\left|\Gamma\left(\frac{3}{4} + \frac{im}{2m_0} + \frac{i\tilde{\mu}}{2}\right)\right|^2 4m^{-1/2}\pi^{-1/2}\sin^{2(m-1)}\theta$$

$$\times(1+m^2\cos^2\theta)\,\delta_{e,|m|+1}\,. \qquad (8)$$

We have introduced here the notation

$$m_0 = \frac{2\sqrt{3}}{x_0\omega_0}E^2, \quad \omega_0 = \frac{2}{\sqrt{x_0}(3+x_0)}, \quad \tilde{\mu} = \frac{1 - \frac{a^2\omega_0^2}{2}}{2x_0\omega_0\sqrt{3}},$$

$$C(x_0) \approx 0.02 \ (a=0), \quad C(x_0) = \frac{\sqrt{3}}{2}, \quad (a=1).$$

The radiation is not isotropic in this case and is concentrated near the equatorial plane in the angle region $\theta = \pi/2 \pm \tilde{\Delta}\theta$, $\tilde{\Delta}\theta \sim m^{-1/2}$. Just as in the nonrelativistic case, as $\alpha \to 1$, corresponding to formulas (7), (8), and (9), we have $x_0 \to 1$, the radiation intensity is substantially decreased when the source approaches the absolute horizon of the events. The factor

$$e^{-\frac{\pi}{2}\left\{\frac{m}{m_0} + [2(l-|m|)+1]\tilde{\mu}\right\}}$$

ensures effective cutoff of the GSR intensity for large harmonics $m > m_0$ and large angles $\tilde{\Delta}\theta$.

2) The separation by polarizations corresponds to separation by parity of the radiated modes: $W_\sigma \sim (-1)^l$, $W_\pi \sim (-1)^{l+1}$. The contributions from the odd modes to W_σ and from even modes to W_π do not exceed several percent. Here l is the total angular momentum of the wave throughout.

3) The relative contributions W_σ and W_π to the total power $W = W_\sigma + W_\pi$ vary little with the parameter α and are equal to

$$\frac{W_\sigma^{a=0}}{W^{a=0}} \approx 94\%, \qquad \frac{W_\sigma^{a=1}}{W^{a=1}} \approx 93\%.$$

The global radiation is practically linearly polarized, but for $m < m_0$ there are angles $\theta_0(m) \approx \pi/2 \pm 1.2m^{-1/2}$ such that $W_\sigma(m, m_0, \theta_0) = W_\pi(m, m_0, \theta_0)$, i.e., at given $\theta_0(m)$ the radiation has circular polarization. Generally speaking, waves propagating at angles $\theta \neq \pi/2$ are elliptically polarized.

4) When an electromagnetic wave propagates at an angle $\theta_1 \neq \pi/2$ in a Kerr field, the plane of polarization of the electromagnetic wave rotates through an angle

$$\beta \sim a\omega_0\left(\cos\theta_1 + \frac{1}{m}\right).$$

5) The global intensity of the GSR is equal to

$$W \approx \frac{ce^2}{r_p^2}\left(\frac{E}{\mu c^2}\right)^2 \text{ at } x_0 \neq 1.$$

6) At $x_0 = 3$, corresponding to $\alpha = 0$, formulas (7), (8), and (9) describe the emission of electromagnetic waves by a relativistic particle in a Schwarzschild field.

1)We use the Kerr metric expressed in the coordinates t, r, θ, and ϕ, in a form given by Boyer and Linquist (see[1]), in a system of units with $G = c = 1$. We consider the so-called right circular orbit in the equatorial plane in the Kerr metric, with radius x_p. The quantities Ω, ω_p, x_p, and x_{hor} are expressed in units of the mass M:

$$\omega_p = \frac{d\phi}{dt}\frac{1}{M}, \quad x_p = \frac{r_p}{M}, \quad x_{hor} = \frac{r_{hor}}{M}.$$

2)The phase velocity of the wave, in the system of a remote observer, is opposed to the group velocity, which is directed in any coordinate system towards the surface of the horizon of events.

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