The π condensate and data on the scattering of electrons by nuclei

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The hypothesis is advanced that the experimentally observed distortion of the scattering curve at momentum transfer $q \simeq 3$ F^{-1} is due to the layered nuclear-matter structure caused by π condensation. Other possible experiments in which the layered structure of nuclear matter might be manifest are discussed.

1. LAYERED STRUCTURE CAUSED BY π CONDENSATION

Condensation means that a layered spin structure of the nucleons is produced in nuclear matter. It is convenient to describe the phenomenon with the aid of a pion field. In unbounded nuclear matter, the condensate pion field is described by[1–3]

$$\phi = \alpha \sin k_0 z. \tag{1}$$

The quantities α and k_0 are determined from the equation for the energy $\omega(k)$ of the pions in matter.[2,3] The layered structure of the spin density leads, in second order in the amplitude, to the layered structure of the density of both the neutrons and the protons

$$n(r) = n^0(r)(1 + \xi^2 \cos 2k_0 z), \tag{2}$$

where

$$\xi^2 = \frac{3}{16}\frac{f^2 k_0^2 \alpha^2}{\epsilon_F^2}; \quad f = \frac{g}{2m} \simeq 1 \quad (\hbar = m_\pi = c = 1).$$

Calculation yields for k_0 a value $k_0 \approx 1.5$ F^{-1}.[3] Solution of the equation for the condensate field in a finite system leads to the following results:[4] In a system of sufficient size, a planar layered structure is established [formula (1)], which goes over in the layer κ into the condition $\phi = 0$ on the surface. If the radius of the system is $R \lesssim \kappa$, then spherical layers $\phi = \alpha \cos k_0 r$ are realized. According to estimates,[4] in medium and heavy nuclei we have $R > \kappa$, and plane layers should be produced. The existence of such layers should lead to a rotational structure of the spectra. If the amplitude of the relative density modulation is small, then the corresponding moment of inertia is small and the energies of the rotational excitations are high. Averaging of the layered structure (2) over the direction of the vector k_0 will then take place in the ground state, and in experiments on elastic scattering there would appear in place of (2) the following density distribution

$$\tilde{n}(r) = n^0(r)\left(1 + \xi^2 \frac{\sin 2k_0 r}{2k_0 r}\right). \tag{3}$$

The density modulation gives rise to the Born form factor; this increment can be estimated (at $q \approx 2k_0$) by the expression

$$\delta F^{(\pi)}(q) = \frac{3\xi^2}{2q^2R^2}\frac{\sin(q - 2k_0)R}{(q - 2k_0)R} \tag{4}$$

$\delta F^{(\pi)}(q)$ has a sharp maximum at $q = 2k_0 \approx 3$ F^{-1}. As is well known, experiments on elastic scattering of electrons by nuclei[5] were analyzed in the following manner. The proton distribution density was chosen in the form

$$n_p(r) = n_p(0)\frac{1 + \frac{wr^2}{R^2}}{1 + e^{\frac{r - R}{\delta}}}. \tag{5}$$

The constants w, R, and δ were chosen such as to obtain the best description of the experimental data in a wide range of small q ($q < 2$F^{-1}), and the constant $n_p(0)$ was determined from the condition $Z = \int n_p dv$. After finding the constants, the cross sections were calculated for large q. The general result of all the experiments was that a large deviation, sometimes by one order of magnitude, from the cross section calculated from the distribution (5) was observed in a narrow interval q near $q = q_0 = 3$ F^{-1}. A similar phenomenon is observed also in proton scattering.[6] Even in this case, a deviation from the calculated cross section obtained with the aid of the optical potential is observed at $q \approx 3$ F^{-1}.

The changes in the cross section occurring in a narrow interval of q apparently point to the existence of a periodic structure of the density, of the type (3), in all the investigated nuclei. Let us examine by way of illustration the Born approximation. The significant part of the Born form factor, corresponding to the distribution (3), is given by (at $qR \gg 1$)

$$F(q) = -\frac{3}{q^2R^2}\left(\cos qR\, \psi(\pi q\delta)\right) - \frac{\xi^2}{2}\frac{\sin(q - 2k_0)R}{(q - 2k_0)R}, \tag{6}$$

where $\psi(x) = x/\sinh x$. The experimentally observed change of F means that at $q \approx q_0$

$$\xi^2 \cong 2\psi(\pi q_0 \delta)\cos q_0 R.$$

The value of ξ^2 obtained from this condition is of the order of 5×10^{-2}. The relatively small scatter of the values of ξ^2 for different nuclei serves as a certain argument favoring our assumption.

2. SHELL FLUCTUATIONS OF THE DENSITY

We shall show that the shell fluctuations of the density can apparently not account for the observed course of the cross section. Shell modulations of the density can be obtained analytically in the quasiclassical approximation (see, e.g., [7]). The corresponding form factor is a smooth function of q. This smooth function is suppressed to a considerable degree in the analysis method used to reduce the experimental data on scattering. Taking a smooth function of q into account leads only to a small change in the empirical constants of the distribution (5).

To check this, the following computational experiment was performed. The distribution of the proton density was obtained for Pb^{208} and Ca^{40} with the aid of the ψ-functions of the individual nucleons in the Woods-Saxon model. This was followed by a Fourier analysis, i.e., by the determination of the Born-approximation form factor corresponding to the obtained density distribution. Then, as is done in the analysis of experiments, parameters giving the best agreement with the form factor at $q < 2\ F^{-1}$ were obtained for the distribution (5), after which the form factor was calculated for $q > 2\ F^{-1}$. The deviations of the form factor corresponding to a smooth distribution from the true form factor are small and are distributed over a wide interval of q.

It should be noted that there exist shell-fluctuation calculations in which, at the cost of introducing an interaction that contains arbitrary parameters, it is possible to account for the cross-section curve at $q \approx 3\ F^{-1}$, but the agreement becomes much worse for smaller q.[9] Our assumption seems to be more natural.

3. POSSIBLE EXPERIMENTS AIMED AT VERIFYING THE LAYERED STRUCTURE OF NUCLEAR MATTER

More complete information on the layered structure could be obtained in experiments on scattering of electrons by oriented nuclei. Experiments of this type were carried out on Ho^{165},[8] but the transferred momenta were too small. In these experiments, the orientation of the nuclei led to orientation of the quadrupole moment, since the odd proton in Ho has a momentum projection on the elongation direction $m = j$ ($j = 7/12$). Since, according to,[4] the direction of the layers is fixed in terms of energy with the elongation direction, the orientation of the nuclei means also orientation of the layers. This can lead to an increase of the diffraction by the layers in comparison with the case of layers averaged over the directions.

We note that experiments on the scattering of polarized electrons would make it possible to observe a layered nuclear magnetic structure corresponding to the periodicity of the spin density of nuclear matter. The related maximum of the scattering curve should then correspond to $q = k_0$, i.e., to half as large a momentum transfer than in the case of scattering by the charge distribution. A layered spin structure could appear also in experiments on the scattering of pions and protons by oriented nuclei.

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